Evidence and Skepticism in Verifiable Disclosure Games[*]

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Abstract

A key feature of communication with evidence is skepticism: a receiver will attribute any incomplete disclosure to the sender concealing unfavorable evidence. I study when a change in the receiver's prior belief about the sender's evidence induces more skepticism, i.e. induces the receiver, regardless of his preferences, to take an equilibrium action that is less favorable for the sender following every message. I provide a definition of when one receiver prior belief expects more evidence than another and show that this characterizes more skepticism. As an input, I fully characterize receiver optimal equilibrium outcomes in general verifiable disclosure games.

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1. Introduction

Communication during criminal trials, financial audits, and investment pitches is often verifiable. In these settings, communication is less about the risk of misrepresentation (cheap talk) and more about which evidence is presented or omitted (disclosure). Any rational observer (receiver) is naturally *skeptical* of the evidence presented by an interested party (sender): the receiver will partially attribute incomplete disclosures to the sender concealing unfavorable evidence.

This skepticism is harmful to the sender: a prosecutor would always prefer to be faced with a less skeptical juror, and an entrepreneur a less skeptical investor. It is also natural that the receiver's beliefs about the sender's evidence modulate his degree of skepticism. Indeed, the criminal justice literature identifies a "CSI effect": Shelton et al. (2009) find that jurors who are more informed about forensics are less likely to convict given the same evidence profile. However, it is not clear which beliefs induce more skepticism than others. The main goal of this paper is to characterize this comparison, i.e. to identify the sender's preferences over the receiver's prior beliefs. Understanding these preferences is important because the sender can often influence the receiver beliefs he faces. For example:

1. During jury selection, a prosecutor questions potential jurors in order to identify those that will return the highest probability of conviction. The prosecutor selects jurors on many criteria including their beliefs about the evidence available. The prosecutor wants to choose jurors who hold the least skeptical beliefs concerning his evidence. Which juror beliefs will achieve this goal?

2. During each investment round, an entrepreneur discloses customer reviews, prototypes, and sales numbers to an investor in order to obtain funding. There are generally multiple rounds of investment. Thus, the entrepreneur will be concerned about how an early disclosure affects the skepticism about future disclosures. Which beliefs will the entrepreneur want to induce in these future rounds, and which early disclosures will achieve this goal?

The common features of these examples is that there is some preliminary action (jury selection) which affects the receiver's beliefs going into verifiable disclosure (e.g. the criminal trial). Thus, a key issue is understanding the sender's preferences over these beliefs within the static verifiable disclosure framework.

One intuition is that the receiver's degree of skepticism increases when he expects more evidence to be available. This accords with the CSI effect: prosecutors will try to avoid jurors with bullish views about the amount of evidence that can be presented. The issue is that there are potentially multiple dimensions over which the sender can be informed, for example a prosecutor can have access to DNA evidence, witness testimony, other forensic evidence, or any subset of these. This makes it difficult to even define what it means to believe the sender has more evidence. Indeed, there do not exist general comparative statics or characterization results in multidimensional verifiable disclosure models.

I study a general verifiable disclosure model in which a sender communicates with a receiver who then chooses an action.[1] While the sender always prefers higher actions, the receiver's preferences depend on the sender's private evidence realization or type. The type space T doubles as the message space, and the messages available to each type are governed by a partial disclosure order $\succeq_d$ on T: if one sender type dominates another according to the disclosure order, i.e., $t' \succeq_d t''$, then the former can make a declaration to the receiver that he is the latter type, i.e., t' can send message t''. This can be interpreted as type t' presenting all the evidence type t'' possesses: a prosecutor type with DNA evidence dominates a prosecutor type with no evidence according to the disclosure order as the former can masquerade as the latter through omission. In Subsection 2.1, I describe how this framework captures familiar examples from the literature.

In the context of arbitrarily complex evidence structures, I provide a definition of when one receiver belief over the sender's type has "more evidence" than another: if one sender type dominates another according to the disclosure order, the former is relatively more likely than the latter under a distribution with more evidence. Focusing on the receiver optimal equilibrium, Theorem 1 establishes that the sender obtains lower actions regardless of his type realization or the receiver's preferences when he is believed to have more evidence. Equivalently, regardless of the true distribution of evidence and the preferences of the receiver he faces, the sender *always* prefers to be thought of as having less evidence. Moreover, the converse is also true: if the sender prefers to induce one receiver belief over another in this broad sense, then the preferred belief must have less evidence.

[1] The receiver chooses an action in $\mathbb{R}$.

My model makes no assumptions on the relationship between the amount of evidence the sender has and its "value", i.e., how high an action it would induce from the receiver if truthfully revealed. Instead the relationship between more evidence and more skepticism emerges from the receiver optimal equilibrium structure. My second main contribution is to fully characterize the receiver optimal equilibrium. Proposition 1 provides necessary and sufficient conditions for a set of sender types to "pool together", i.e. obtain the same equilibrium outcome. This leads to an explicit expression for the receiver optimal equilibrium allocation in Theorem 2. At a high level, the pooled set for a given sender type forms through simultaneously minimizing the receiver's value over types that choose to mimic him and maximizing the receiver's value over the types that he chooses to mimic.

Signaling to affect receiver beliefs and decrease skepticism is at the heart of many dynamic disclosure papers. Indeed, Section 3 discusses how previous studies use examples of more evidence changes in the Dye evidence model as a focal point of their analysis. Theorem 1 can be used to generalize these insights as well as to answer questions that necessitate more complicated evidence structures.

The application in Section 6 is a proof of concept. I ask whether an investor benefits from early communication with an entrepreneur who obtains evidence gradually, rather than communicating only right before the investment decision. Specifically, I add an additional period to the static verifiable disclosure game before the receiver's action choice during which the sender can accumulate and disclose evidence. The potential for informative early disclosures relies on the sender's uncertainty about his final evidence and so this question is moot in the Dye model where there is no residual uncertainty after any evidence is disclosed. Proposition 5 shows that the receiver does not benefit from early communication regardless of his preferences or prior beliefs if and only if the evidence structure satisfies what I term the "Unique Evidence Path Property" (UEPP). The interpretation of the UEPP is that the current evidence reveals the sequence of previous investigations undertaken, e.g. the performance test results for a prototype can only be revealed if the prototype is first developed.

Layout Subsection 1.1 discusses the related literature. Section 2 presents the model and lists examples that fit my framework. Section 3 defines the more evidence order and states the main result. Section 4 and Section 5 provide the analysis necessary to establish the main result. Section 6 studies a dynamic disclosure application. All proofs are in the appendix.

1.1. Related Literature

The first verifiable disclosure models were introduced by Milgrom (1981), Grossman (1981), and Grossman and Hart (1980). In these models, the sender could be vague about his private information but not lie, i.e. he could declare any subset of states that contains the true state. The main finding is the "unraveling" result that in any equilibrium the sender fully reveals his information. There are many ways unraveling can fail: if the sender's direction of bias depends on his type (e.g. Seidmann and Winter (1997)), if the sender pays a cost to disclose information (e.g. Verrecchia (1983)), or if the receiver is uncertain about the sender's information endowment which is the focus of this paper (e.g. Dye (1985) and Jung and Kwon (1988)).[2,3]

In line with this paper, various studies allow for more general evidence structures, but instead focus on establishing that the receiver's utility in some equilibrium of the verifiable disclosure game is the same as that in which the receiver can commit to a best response before learning the sender's message. This equivalence was first introduced in Glazer and Rubinstein (2004) and further explored by Sher (2011), and Ben-Porath et al. (2017) in the context of multiple senders. Hart et al. (2017) identifies the equilibrium that achieves this equivalence through the "truth leaning refinement". I focus on this receiver optimal equilibrium and my model is the same as that in Hart et al. (2017).

In this general verifiable disclosure model, my focus is (i) equilibrium characterization, and (ii) understanding determinants of the receiver's skepticism, or more concretely, comparative statics on the receiver's beliefs. Shin (2003) and Dziuda (2011) characterize and analyze equilibria in multidimensional versions of the Dye evidence framework with the simplification that each piece of evidence is either

[2] Hagenbach et al. (2014) and Mathis (2008) provide necessary and sufficient conditions for unraveling in a general framework.

[3] For surveys of this literature see Milgrom (2008) and Dranove and Jin (2010).

"good" or "bad".[4] Given mild assumptions on the distribution, the sender always plays a "sanitization" strategy of concealing bad pieces and fully revealing good pieces. This means that the decision of whether to disclose a given piece of evidence does not depend on other evidence possessed by the sender. This independence is not general to multidimensional evidence models. Sher (2014) and Glazer and Rubinstein (2004) derive methods to find the receiver optimal equilibrium with more general evidence structures but restrict attention to the receiver choosing between two actions, and the case where the sender has two payoff relevant types - acceptable and unacceptable. Bertomeu and Cianciaruso (2016) propose an algorithm for solving verifiable disclosure games when pure strategy equilibria exist. My approach focuses on equilibrium outcomes instead of sender strategies. This permits tractable analysis despite the fact that sometimes (generically) verifiable disclosure games only admit mixed strategy equilibria.

Comparative statics results on the receiver's skepticism have mostly been limited to the Dye evidence model. In particular, dynamic disclosure models such as that in Guttman et al. (2014), Acharya et al. (2011), and Grubb (2011) develop these conclusions as the backbone of their analyses. The idea is that all else held equal, the sender will take decisions in early periods that engender less skepticism in the future. As Section 3 elaborates, these results are specific cases of Theorem 1.

2. Model

The setting involves a single sender and a single receiver. The sender privately observes his type $t \in T$, where $|T| = n$, and sends a message to the receiver who chooses an action $a \in A \equiv \mathbb{R}$. The receiver has a prior belief $h \in \Delta T$ over the sender's type with associated measure $H : 2^T \to [0, 1]$.[5] The sender may have some alternative prior belief over his type but it is not relevant to the results.

Preferences The sender's utility, $U^S : A \times T \to \mathbb{R}$, is strictly increasing in the action a for every type t.[6] The receiver's utility, $U^R : A \times T \to \mathbb{R}$, depends on both

[4] Dziuda (2011) also considers uncertainty over the direction of monotonic preferences of the sender and over whether he is honest or strategic.

[5] I refer to distributions with lower case and their associated measures with upper case.

[6] In section 7 of my earlier working paper Rappoport (2020), I show that the analysis adapts readily to the case where the sender has a (potentially evidence dependent) probability of preferring lower actions or having the same preferences as the receiver.

the action and the sender's type. I assume that $\forall t \in T,\ U^R(\cdot, t)$ is strictly concave, continuously differentiable, and admits a maximum. I denote the set of all such receiver utilities as Υ. These assumptions imply that neither party will randomize over induced actions in equilibrium, and so to ease notation I identify the sender's utility with the action taken, i.e. $U^S(a,t) \equiv a$.[7]

For each $t \in T$, denote $\boldsymbol{v(t)} \equiv \arg\max_a U^R(a,t)$. Similarly, define $\boldsymbol{V_h(S)} \equiv \arg\max_a \mathbb{E}[U^R(a,t)|t \in S, t \sim h]$ to be the receiver's best response conditional on the sender's type being in S and distributed according to h. I refer to sets of types with relatively high (low) optimal actions, as "high (low) value". A leading example is when the receiver's utility is quadratic loss, i.e. $U^R(a,t) = -(a - v(t))^2$ for some function $v : T \to \mathbb{R}$. In this case $V_h(S) = \mathbb{E}[v(t)|t \in S, t \sim h]$.

Messaging Technology I follow Hart et al. (2017) and assume that the message space is the type space with the interpretation that type t sending message t' is type t "mimicking" t'. I assume that there is a partial **disclosure order**, $\boldsymbol{\succeq_d}$, over T. The relation $t \succeq_d t'$ means that t can send message t', i.e., the set of available messages to each type t is given by $\{t' : t \succeq_d t'\}$.[8] The partial order assumption imposes reflexivity, transitivity, and antisymmetry. That is, (i) t can send message t, (ii) if t can mimic t' and t' can mimic t'', then t can mimic t'', and (iii) for $t \neq t'$, t can mimic t' implies t' cannot mimic t.[9]

This means that the sender's type specifies two pieces of information: its position in the disclosure order, and its best response to the receiver. Importantly, my model makes no assumption on the relationship between these two aspects.

[7] Strict concavity ensures that the receiver has a unique best response for all distributions $h \in \Delta T$. In combination with the assumption that the sender's utility is strictly increasing in a for all t, this implies that the sender will never randomize over messages which induce different actions.

[8] The verifiable disclosure literature sometimes uses an alternative equivalent set of messaging assumptions. There is an arbitrary message space M and each type has access to some subset $E(t) \subset M$, where the message correspondence E satisfies a "normality" (Bull and Watson (2004)) or "nested range" (Green and Laffont (1986)) condition. A message structure is normal if $\forall t \in T$, there exists $e_t \subset E(t)$, such that $\forall t, t' \in T$, $e_t \in E(t') \implies E(t) \subset E(t')$. Given a normal message structure, the following disclosure ordered type space $(T, \succeq_d)$ has the same set of equilibrium allocations. $T \equiv M$ with $\mathbb{P}(e_t) \equiv \mathbb{P}(t)$, $\mathbb{P}(m) = 0$ otherwise, and $e_t \succeq_d m \iff m \in E(t)$.

[9] Antisymmetry is without loss in the following sense. Consider that in addition to disclosable evidence from $(T', \succeq_d)$, the sender also has private information $\theta \in \Theta$ so the type space is $T \times \Theta$. Message feasibility is given by the *preorder* $\succeq_d'$ defined by $(t, \theta) \succeq_d' (t', \theta') \iff t \succeq_d t'$. Because the sender always prefers higher actions, (t, θ) must induce the same equilibrium action $\forall \theta \in \Theta$.

Strategies and Equilibrium A strategy for the sender is $\sigma : T \to \Delta T$ where $\text{Supp}(\sigma_t) \subseteq \{t' : t \succeq_d t'\}\ \forall t$. Because the receiver's utility is strictly concave, it is without loss to restrict the receiver to use a pure strategy, $a : T \to A$, which specifies an action choice in response to each message. A perfect Bayes equilibrium (PBE) is a pair of feasible strategies (σ, a) such that $\forall t \in T$,

$$(i) \qquad \text{Supp}(\sigma_t) \subseteq \underset{t':\, t \succeq_d t'}{\arg\max}\ a(t'),$$

$$(ii) \qquad a(t) = \underset{a \in A}{\arg\max}\, \mathbb{E}[U^R(a,s)|\sigma,t] \text{ if } t \in \cup_{t \in T} \text{Supp}(\sigma_t), \text{ and}$$

$$(iii) \qquad a(t) = V_q(\{t' : t' \succeq_d t\}) \text{ for some } q \in \Delta\{t' : t' \succeq_d t\}.$$

In words: (i) the sender maximizes the receiver's best response among feasible messages, (ii) for on-path messages, the receiver updates according to Bayes rule and best responds, and (iii) for off path messages, the receiver best responds to some belief over sender types that have access to the message.

I focus on the receiver optimal PBE which I henceforth refer to as the **ROE**. Denote $\boldsymbol{\pi_h(t|U^R)} \in A$ as the ROE allocation when the receiver has prior belief $h \in \Delta T$ and utility function U^R, and the sender is type $t \in \text{Supp}(h)$. ROE strategies are not unique and are kept in the background of the analysis.[10]

2.1. Examples

Common disclosure models that fit my framework are described below. I illustrate $(T, \succeq_d)$ as a directed graph with vertices representing types and directed paths representing all non-reflexive dominance in the disclosure order.

Dye Evidence: The Dye evidence model was introduced by Dye (1985) and Jung and Kwon (1988). The type space is given by $T \equiv \{t_\emptyset, t_1, ..., t_{n-1}\}$, where $t_1, ..., t_{n-1}$ represent different evidence realizations, and $t_\emptyset$ represents the case where the sender has no evidence. The disclosure order is given by $t_i \succeq_d t_\emptyset\ \forall i = 1, ..., n-1$ with no

[10] As discussed in Section 4, unrefined PBE admit multiplicity and put very little structure on equilibrium outcomes. A number of studies have provided justifications for focusing on the ROE. Hart et al. (2017) shows that the truth leaning refinement, in which (i) the receiver interprets each off path message credulously, and (ii) the sender is truthful when doing so maximizes his obtained action, selects the ROE. Hart et al. (2017) also show that perturbations where the sender has small probability of being honest have a unique equilibrium outcome that is arbitrarily close to the ROE in the limit. In addition, Sher (2011) and Hart et al. (2017) establish that the ROE is equivalent to outcomes in the case where the principal can commit.

other non-reflexive relations. Figure 1 illustrates the disclosure order in the Dye model. The interpretation is that the sender who obtains evidence t_i must either fully reveal his type or completely withhold, i.e., declare $t_\emptyset$; accordingly, $t_\emptyset$ is unable to verify that he is uninformed. In this sense, the Dye evidence model is distinguished by "all or nothing" disclosure which greatly simplifies its analysis.

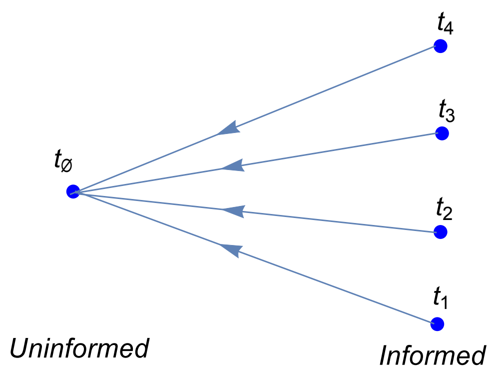


Figure 1: Dye Evidence with $n = 5$.

Multidimensional Evidence: Suppose there are k potential kinds of evidence each drawn from $E \equiv \{e_1, ..., e_m\}$. Depending on the application, there are different ways to conceive of multidimensional evidence and the corresponding disclosure order. First, consider that each type t is an $l \leq k$ sample of E with replacement, and $\bar{t} \succeq_d \underline{t}$ if and only if each $e_i \in \underline{t}$ appears with greater frequency in $\bar{t}$.[11] That is, a type mimics another by withholding the residual evidence. I term this multidimensional evidence structure as **Independent Collection**. For example, an entrepreneur discloses a subset of his customer reviews which range from 0 to 5 stars.[12,13] An example is displayed in the left panel of Figure 2.

[11] More formally, each type is a function $t : E \to \mathbb{N}$, where $\sum_i t(e_i) \leq k$. $t(e_i)$ encodes how many draws of evidence e_i are possessed by type t. Accordingly $t_1 \succeq_d t_2 \iff t_1(e_i) \geq t_2(e_i)\ \forall i$.

[12] In the independent collection evidence structure evidence is *indistinguishable*. However, one can encode distinguishable evidence through the support of h as follows. For $i \leq k$, let $E^i \equiv \{e^i_1, ..., e^i_{m_i}\}$ represent the set of available evidence from source i. Construct an independent collection disclosure order where $E = \cup_i E^i$ and a distribution over T that puts zero mass on any sample that includes multiple draws from the same E_i. This distinguishability is implicit under sequential collection, as displayed in the right panel of Figure 2.

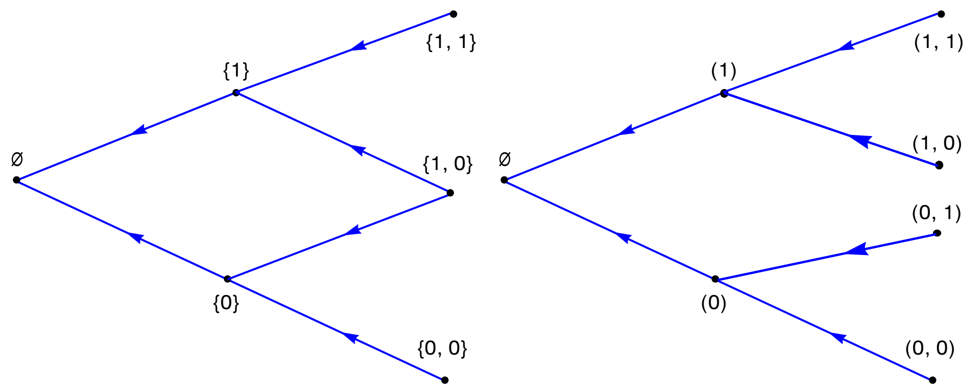
[13] A related disclosure order is that used in "vagueness" models, e.g., that in Milgrom (1981) and Hagenbach et al. (2014). These models feature a state $x \in X$ and a set of messages (and therefore types in the current modeling) given by the subsets of states $2^X \equiv T$. The disclosure order is given

An alternative natural multidimensional disclosure order is as follows. A type is an *ordered* subset $(r_1, ..., r_l)$ where $0 \leq l \leq k$ and each $r_i \in E$. For two types $\bar{t} \equiv (\bar{r}_1, ..., \bar{r}_{\bar{l}})$ and $\underline{t} \equiv (\underline{r}_1, ..., \underline{r}_{\underline{l}})$, $\bar{t} \succeq_d \underline{t}$ if and only if $\bar{l} \geq \underline{l}$ and $\bar{r}_i = \underline{r}_i \; \forall i \leq \underline{l}$. This captures the idea that evidence collection is sequential, and accordingly, I term this multidimensional evidence structure **Sequential Collection**. For example, a prosecutor's investigation can find a potential suspect and then check for a verifiable refutation of their alibi, but revealing information about the alibi reveals a suspect was found in the first place. Alternatively, there is a natural truncation structure to the support of the evidence distribution. For example, an applicant can choose the date to start reporting work experience on their CV, but unemployed segments after are "resume gaps". The right panel of Figure 2 presents an example of a sequential collection disclosure order.

Multidimensional evidence potentially adds two aspects to the all or nothing disclosure decision in the Dye model. Under sequential collection, the sender decides "how much" evidence to disclose: in the right panel of Figure 2, type $(0, 1)$ decides whether to report no results— $t_\emptyset$, the results of his first investigation — (0), or the results of both — $(0, 1)$. Under independent collection, the sender not only decides how much, but also "which" evidence to disclose: in the left panel of Figure 2, given that type $\{1, 0\}$ makes a partial disclosure they must still decide between $\{0\}$ and $\{1\}$.

Honest Types: In addition to obtaining evidence from some T', the sender can either be strategic, S, or honest, H. Strategic types can disclose evidence according to some arbitrary disclosure order $\succeq_d'$, while honest types must truthfully reveal. The total type space and disclosure order are denoted by $(T, \succeq_d)$ defined as follows: $T \equiv T' \times \{S, H\}$ and $t \succeq_d' t' \implies (t, S) \succeq_d (t', \theta') \; \forall \, t, t' \in T', \; \forall \theta' \in \{S, H\}$ with $\succeq_d$ admitting no other non-reflexive relations. Figure 3 displays the Dye model with the addition of honest types.

by $\bar{t} \succeq_d \underline{t}$ if and only if $\bar{t} \subseteq \underline{t}$. However, this is isomorphic to a multidimensional independent collection disclosure order defined as follows: $E = X$, and each subset $t \subset X$ is mapped to the type $\tilde{t}(e_i) = \mathbb{1}_{e_i \not\in t}$, and the prior belief h is such that $t(e_i) > 1$ for some i implies $h(t) = 0$. More noteworthy is that vagueness models commonly assume the sender "knows" the state, i.e., the distribution of types h is supported only on the singleton elements of $2^X = T$. This demonstrates how zero probability types capture extra messaging options for the sender.



(a) Independent Collection. (b) Sequential Collection.

Figure 2: Examples of multidimensional evidence for $E = \{0, 1\}$ and $k = 2$.

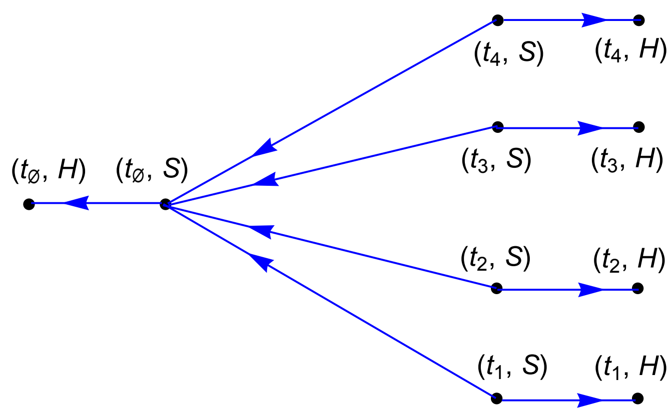


Figure 3: An honest types model with $(T', \succeq'_d)$ as Dye evidence

3. Characterizing Increased Skepticism

The main goal of this paper is to understand what drives receiver "skepticism", that is, fixing the disclosure order, how does $\pi_h(t|U^R)$ depend on the receiver's prior belief h? I focus on the effects of the receiver's beliefs that do not depend on his particular preferences; it is natural that increasing the probability of higher valued types will increase equilibrium actions, but such a change is not naturally interpreted as a decrease in skepticism. Theorem 1 below characterizes which beliefs induce higher equilibrium actions regardless of the receiver's preferences. The key ingredient is the definition below.

Definition 1. Let $f, g \in \Delta T$ be two receiver prior beliefs over the sender's evidence. f has **more evidence** than g with respect to $(T, \succeq_d)$, or $f \geq_{ME} g$, if

$$\forall t, t' \in T, \quad t \succeq_d t' \implies f(t)g(t') \geq f(t')g(t). \tag{1}$$

For any type t that can mimic t', t is relatively more likely than t' under a prior distribution with more evidence. With the view that each type is a set of evidence which the sender can present, as in the independent collection structure from Subsection 2.1, the more evidence relation shifts probability to types that literally have more evidence in a subset containment sense. It is worth noting that the more evidence order is silent on whether one receiver belief is more "optimistic" than another: the dominating types according to the disclosure order can be higher or lower value relative to dominated types depending on the receiver's preferences.[14]

If $\succeq_d$ were a complete order, then $f \geq_{ME} g$ would be equivalent to f monotone likelihood ratio (MLR) dominates g on $(T, \succeq_d)$. Definition 1 is an extension of MLR dominance to a partially ordered set that only imposes the likelihood ratio inequality on comparable pairs of types.[15] The interpretation is that the more evidence relation places no restriction on the relative probability of different kinds of evidence, e.g. a distribution with more evidence can decrease or increase the relative probability of DNA evidence to witness testimony.

Theorem 1. *Let $f, g \in \Delta T$ with $I \equiv \text{Supp}(f) \cap \text{Supp}(g)$. If $f \geq_{ME} g$, then,*

$$\pi_f(t|U^R) \leq \pi_g(t|U^R) \quad \forall t \in I, \ \forall U^R \in \Upsilon. \tag{2}$$

If f and g have the same support and condition (2) holds, then $f \geq_{ME} g$.

[14] In this sense, a more evidence change captures more than just changes in the information structure of the sender about a payoff relevant state with a fixed prior; e.g., more evidence changes can capture increases in the "average value" of the sender's evidence. On the other hand, because the value of each type to the receiver is assumed to remain constant across changes in the distribution of evidence, there are changes in the information structure that do not correspond to changes in the distribution of evidence in the current model. In my earlier working paper Rappoport (2020), section 7.3 develops a more universal approach that captures all changes in the information structure.

[15] There are existing notions adapting the likelihood ratio order to a partially ordered set. The most common version is the multivariate likelihood ratio order described in chapter 6.E of Shaked and Shanthikumar (1994) which is stronger than Definition 1 because it restricts to lattice orders, and imposes likelihood restrictions on incomparable elements. My notion has been raised in previous studies (e.g., definition 1 in Whitt (1982)), but does not appear to be frequently used.

The result says that if f has more evidence than g, every type obtains a lower equilibrium action when facing a receiver who holds beliefs g than one who holds beliefs f, and that this comparison does not depend on the receiver's preference. If f and g have common support, then the converse also holds.[16] The more evidence relation exhausts the sense to which one can compare equilibrium actions based on the receiver's beliefs alone. That is, the mechanism behind Theorem 1 is increased *skepticism*: the receiver believes that any equilibrium message is more likely to be the result of strategic withholding on the part of the sender.

The rest of this section is organized as follows. I first discuss how the equivalence result can be interpreted and used in applications. Next, I provide a preview of the analysis and intuition for Theorem 1. Finally, I conclude this section by revisiting Subsection 2.1 to explore the implications of Theorem 1.

Using the Equivalence Result Many natural changes correspond to a more evidence change in the receiver's prior belief: advances in forensics permit testing on a larger fraction of collected samples from crime scenes, or a firm develops a reputation for keeping detailed records and accounts. Indeed, Theorem 1 rationalizes the "CSI effect" mentioned in the introduction whereby potential jurors who expect evidence to be more widely available convict less often.

Beyond these descriptive implications, Theorem 1 provides a key tool in analyzing situations in which the sender can signal or otherwise affect the beliefs of the receiver before disclosing evidence. The corollary below develops this idea by reinterpreting Theorem 1 as a characterization of the sender's preferences over receiver beliefs.

Corollary 1. *Let $f, g \in \Delta T$ with full support. Let $\eta \in \Delta T$ be the sender's actual distribution over evidence. For every $U^R \in \Upsilon$, and $\eta \in \Delta T$, the sender has a higher equilibrium expected utility when facing a receiver who holds beliefs g than one who holds beliefs f if and only if f has more evidence than g.*

It is important to distinguish the above corollary from welfare comparative statics in a common prior model. If both the sender's and receiver's prior beliefs shift

[16] The caveat of full support in the equivalence is because the more evidence definition imposes restrictions outside of the supports of f and g and these are not always relevant to the ROE actions. A more complete converse is established in the proof: if $f(t)g(t') < f(t')g(t)$ for some $t \succeq_d t'$ such that $t' \in \text{Supp}(f) \cap \text{Supp}(g)$ then $\exists U^R \in \Upsilon$ such that $\pi_f(t'|U^R) > \pi_g(t'|U^R)$.

between two distributions, then while it is still true that the sender does better "type by type" under the distribution with less evidence, his ex-ante expected utility can be higher under the distribution with more evidence.[17] Instead the comparison in Corollary 1 is the relevant one for signaling interactions before the disclosure game: when comparing two signals, the sender effectively manipulates the receiver's belief about his evidence, while holding his true distribution over evidence constant. Which juror beliefs should a prosecutor try to select for at *voir dire*? How does an entrepreneur disclosing early development progress affect investor demands for future progress?

This paper is not the first to acknowledge how the sender's preference over receiver beliefs is important in dynamic disclosure. Grubb (2011), Acharya et al. (2011), and Guttman et al. (2014) have used predecessors of Theorem 1 to derive important insights.These papers focus on the one dimensional Dye evidence model of Subsection 2.1 and study specific changes in the evidence distribution that, as Subsection 3.1 elaborates, are examples of more evidence shifts.

With regard to dynamic disclosure, Theorem 1 can be used in two ways to enhance our understanding. First, it can elucidate how to generalize insights to multidimensional evidence structures: Corollary 1 immediately suggests that, all else held equal, prosecutors will select jurors who believe them to have less evidence, regardless of how complicated the evidence structure is. Second, Theorem 1 can answer new questions that are only relevant to multidimensional evidence structures. In this regard, the application in Section 6 is a proof of concept.

An (Incomplete) Intuition and Analysis Road Map The difficult direction in proving Theorem 1 is that more evidence implies lower equilibrium actions. Note that equilibrium behavior consists of certain subsets of sender types, "pooled sets", obtaining the same action from the receiver. The basic idea behind behind the result is that the value of these "pooled" subsets decreases under a more evidence shift in the distribution of types. Because there are no restrictions on the receiver's preferences, the value of an *arbitrary* subset of types can go up or down with a more evidence shift. Thus, the key steps in establishing the main result are to (i)

[17] For example, suppose that the disclosure order is empty, i.e., $\succeq_d$ admits no non-reflexive relations, with the interpretation that every type can verifiably distinguish themselves from all other types. The ROE has full separation, the actions do not depend on the distribution, and $f \geq_{ME} g \; \forall f, g \in \Delta T$, which means that it is possible that $\mathbb{E}[v(t)|t \sim f] \geq \mathbb{E}[v(t)|t \sim g]$.

understand what is special about pooled subsets in the ROE, and (ii) connect these properties with the effects of a more evidence shift in the receiver's beliefs.

To preview the analysis, consider the following compelling, but ultimately incomplete, intuition. Sender types that mimic other types in equilibrium are both more dominant in the disclosure order (have the ability to mimic) and have lower value to the receiver (have the incentive to mimic). Because a more evidence change shifts probability to more dominant types in the disclosure order, in equilibrium, it also shifts probability to lower value types, thereby decreasing the receiver's best response. Indeed, this intuition is correct in the Dye evidence model.

Example 1. For illustration, consider a prosecutor (the sender) disclosing evidence to attempt to persuade a representative juror (the receiver) to convict a defendant for a crime committed sometime between 8 and 11 am. Suppose that the investigation could potentially turn up a witness – A, who was at the crime scene around 8 am (but not after), and reports whether they saw the crime being committed or not. The type space is $T = \{t_\emptyset, A+, A-\}$ indicating that witness A did not make a usable statement, they saw the defendant, and they did not see the defendant respectively. The receiver has quadratic loss and $v(A^+) > v(t_\emptyset) > v(A^-)$. For any receiver beliefs, the ROE involves type A^+ disclosing, and A^- withholding to pool with $t_\emptyset$. Now consider two receiver beliefs $f, g \in \Delta T$, such that $f \geq_{ME} g$. In order to confirm Theorem 1 we only have to look at the value of the pooled set $\{t_\emptyset, A^-\}$. Indeed f moves probability up the disclosure order from $t_\emptyset$ to A^- relative to g, and so because $v(A^-) < v(t_\emptyset)$ it holds that $V_f(\{t_\emptyset, A^-\}) \leq V_g(\{t_\emptyset, A^-\})$.

Now consider the following simple multidimensional extension of this evidence structure. There is another witness – B, who was at the crime scene around 11 am (but not before), and also reports whether they saw the crime being committed or not. For the sake of keeping illustrations simple, suppose that B can only be potentially sought out if A has first made a statement, so that this corresponds to a sequential collection multidimensional evidence structure from Subsection 2.1. The type space is now $\{t_\emptyset, A^+, A^-, A^+B^+, A^-B^-, A^+B^-, A^-B^+\}$, where B^+ and B^- indicate that witness B reported seeing and not seeing the defendant respectively, whereas A^+ and A^- indicate that B did not make a usable statement. The receiver's preferences and the disclosure order are displayed in Figure 4. The prosecutor's

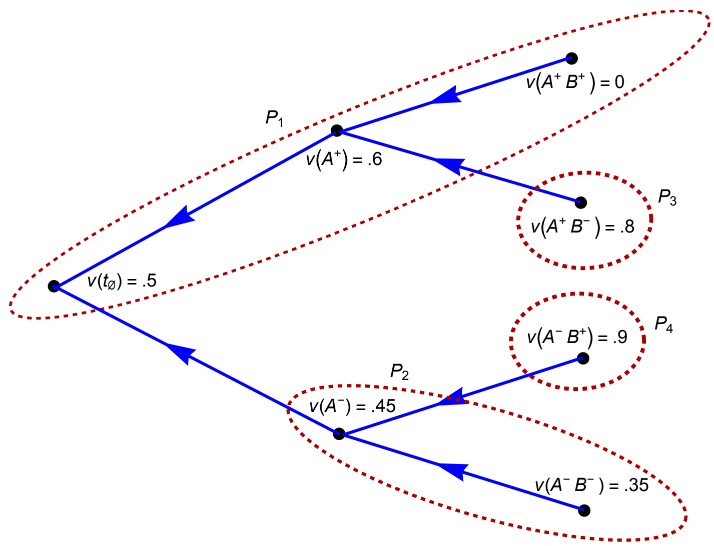


Figure 4: An example where $v(\cdot)$ is not decreasing in $\succeq_d$ within each pooled set.

case is stronger with a single positive identification than a negative identification or unusable testimony. However, two positive identifications, i.e., A^+B^+, is bad for the prosecution's case as two witnesses reporting they saw the crime being committed at different times gets the case thrown out.

Now, consider two distributions, $f, g \in \Delta T$ where g is the uniform distribution over T, $f \geq_{ME} g$, and, for simplicity, f is a "small deviation" from g.[18] The dotted ellipses in Figure 4 display the equilibrium pooled sets under both f and g.[19] In particular, the lowest payoff pooled set involves both types A^+B^+ and A^+ mimicking $t_\emptyset$. Theorem 1 shows that $\pi_f(t|U^R) \leq \pi_g(t|U^R)$ $\forall t \in T$. This means that the value of $P_1 = \{A^+B^+, A^+, t_\emptyset\}$ is lower under f than g. However, unlike in the Dye model, the receiver's value is not decreasing in the disclosure order within P_1, and so we cannot appeal to the same intuition. Relative to g, f shifts probability from A^+ to A^+B^+ which decreases the receiver's value, but also shifts probability from $t_\emptyset$ to A^+ which increases the receiver's value. While this example has been constructed to be simple and isolate the non-monotonicity, such patterns occur regularly as part of pooled sets in multidimensional evidence structures. $\triangle$

[18] For example, let $f(t) \in [g(t) - \varepsilon, g(t) + \varepsilon]$ for small $\varepsilon > 0$. This assumption guarantees that the equilibrium pooled sets are the same between f and g.

[19] Each type declares the least dominant type according to $\succeq_d$ within their pooled set: every $t \in P_1$ declares $t_\emptyset$, every $t \in P_2$ declares A^-, and A^+B^- and A^-B^+ truthfully declare.

In order to establish Theorem 1 we need to understand the structure of the ROE pooled sets, e.g., what is special about the set $\{t_\emptyset, A^+, A^+B^+\}$ in the example above? Proposition 1 identifies this property which also leads to an algorithm for constructing the ROE and an explicit expression for ROE actions in Theorem 2. In Section 5 I show that this defining feature of pooled sets also characterizes when the value of a set decreases under any more evidence shift.

3.1. More Evidence and More Skepticism: Examples

Dye Evidence Model In the Dye evidence model,

$$f \geq_{ME} g \iff f(t_\emptyset)g(t_i) \leq f(t_i)g(t_\emptyset) \quad \forall i = 1, ..., n-1. \tag{3}$$

One receiver belief has more evidence than another, and is thereby less preferred by the sender, if and only if the probability of each evidence type has increased relative to the no evidence type. The more evidence relation imposes no restrictions on the relative probability of evidence types even though these types may pool together by withholding.

Various well known comparative statics are examples of the comparison in (3). For example, many Dye evidence models parameterize the distribution over evidence as follows: the sender obtains no evidence, i.e., is $t_\emptyset$, with probability $1-p$, and obtains evidence t_i with probability $p\tilde{h}(t_i)$ for $\tilde{h} \in \Delta\{t_1, ..., t_{n-1}\}$, and $p \in (0,1)$. Jung and Kwon (1988) observed that increasing p while holding $\tilde{h}$ constant decreases the action for non-disclosure. It is easy to check that this change satisfies the comparison in (3) and thereby corresponds to a more evidence change. An immediate implication is that, in a dynamic disclosure framework where the final period is Dye disclosure, the sender will prefer actions in earlier periods that signal he has a low probability p of obtaining evidence in the future. This is a key result used in the dynamic disclosure papers Grubb (2011) and Acharya et al. (2011).

The key comparative static in Guttman et al. (2014) also pertains to the Dye evidence model. For a fixed evidence distribution $h \in \Delta T$, they condition on a set of evidence types $S \subset \{t_1, ..., t_{n-1}\}$ and the no evidence type to obtain $h_{|S}(t)$.[20]

[20] Formally this is defined as $h_{|S}(t) \equiv \frac{h(t)}{H(S\cup\{t_\emptyset\})}\mathbb{1}_{t\in S\cup\{t_\emptyset\}}$. This emerges naturally as an object of interest in their model because they focus on pure strategy equilibria and so the set of evidence types that make a given disclosure in period 1, which is what the receiver conditions on in period 2, will be a subset of evidence types.

Their backbone result finds that $h_{|S'}$ induces a higher non-disclosure action than $h_{|S''}$ for $S' \subset S''$. Again, it is straightforward to check that these two distributions are comparable according to (3) and so this conclusion is an example of Theorem 1.

Multidimensional Evidence Theorem 1 shows how certain comparative statics results from the Dye model generalize to multidimensional evidence structures. Consider parameterizing the evidence distribution as follows. Fix the distribution of each piece of evidence $i \in 0, 1, ..., k$ as $\tilde{h}_i \in \Delta E$. The number of evidence pieces is drawn from $\eta \in \Delta\{0, 1, ..., k\}$.[21] Call the induced distribution over evidence h_η. It is straightforward to check that if η' monotone likelihood ratio (MLR) dominates η'', i.e., $\eta'(i)\eta''(j) \geq \eta'(j)\eta''(i)\ \forall i \geq j$, then $h_{\eta'}$ has more evidence than $h_{\eta''}$.

This comparison generalizes the above Dye evidence comparative static on p. The fact that Theorem 1 is a characterization of when beliefs are worse for the sender ((2)) reveals that other natural generalizations would not work. In particular, if η' FOSD η'' but η' does not MLR dominate η'', then there exist receiver preferences and an evidence realization such that the sender does worse when the receiver believes he has evidence distributed according to η'' than according to η'.

Honest Types A natural intuition is that the strategic sender does better if he is thought to be honest with higher probability. The idea is that evidence realizations that would be withheld by a strategic sender to mimic some type t, are declared by an honest sender. This in turn makes the receiver treat the sender more favorably when he declares that he is type t. While this intuition has been confirmed with specific evidence structures, Theorem 1 delivers this result generally.

Parameterize the distribution of evidence as follows. Let the probability of an honest and strategic sender be $p, 1-p \in (0,1)$ respectively. Let the distribution over disclosable evidence conditional on the nature of the sender be $\tilde{h}_\theta \in \Delta T'$ for $\theta \in \{S, H\}$. Denote the distribution over the total type space as $h_p \in \Delta T = \Delta(T' \times \{S, H\})$. It is straightforward to check that for $p' \geq p''$, $h_{p''}$ has more evidence than $h_{p'}$. By Theorem 1, the sender does better when facing any receiver who holds beliefs $h_{p'}$ as compared with $h_{p''}$.

[21] In the independent collection case, the l pieces are sampled uniformly without replacement, and in the sequential collection case, the sender obtains evidence with indices $\{0, 1, ..., l\}$.

4. Equilibrium Characterization

The equivalence in Theorem 1 relies on the structure of the ROE. This section characterizes this structure and provides two ways to find the corresponding equilibrium actions. Since I focus on a single distribution in this section, it is without loss to assume a full support receiver prior belief $h \in \Delta T$.

The following notation is useful. For $\tilde{S} \subset T$, let $\boldsymbol{W(\tilde{S})} \equiv \{t \in T : \exists s \in \tilde{S},\ s \succeq_d t\}$, and $\boldsymbol{B(\tilde{S})} \equiv \{t \in T : \exists s \in \tilde{S},\ t \succeq_d s\}$. $W(\tilde{S})$ is the set of types that can be mimicked by some type in $\tilde{S}$, and $B(\tilde{S})$ is the set of types that can mimic some type in $\tilde{S}$. The subset $\tilde{S} \subset S$ is a **lower** (respectively **upper**) **contour subset** of S if $W(\tilde{S}) \cap S = \tilde{S}$ (respectively $B(\tilde{S}) \cap S = \tilde{S}$).

4.1. Equilibria as Partitions

Let $\pi : T \to \mathbb{R}$ be some (not necessarily receiver optimal) equilibrium allocation with $\pi(T) = \{\pi_1, ..., \pi_k\}$ with $i < j \implies \pi_i < \pi_j$. Denote the equivalence classes induced by π as $P_i = \{t \in T :\ \pi(t) = \pi_i\}\ \forall i = 1, ..., m$. I refer to $P = (P_1, ..., P_m)$ as an *equilibrium partition* and each P_i as an equilibrium *pooled set*. While the allocation clearly pins down the equilibrium partition, an equilibrium partition also pins down the allocation. Because each $t \in P_i$ is assigned the same action π_i, this action must be the receiver's best response to the set as a whole, i.e. $\pi_i = V_h(P_i)$. Saying that $P = (P_1, ..., P_m)$ is an equilibrium partition thereby means that there is an equilibrium which allocates $V_h(P_i)$ to every $t \in P_i$ with $V_h(P_i)$ is increasing in i.

Any candidate equilibrium partition must respect the sender's incentives. That is if $t' \succeq_d t''$ then $\pi(t') \geq \pi(t'')$, i.e. $t' \in P_i$, $t'' \in P_j \implies i \geq j$. I call a partition satisfying this property an **interval partition**. Each part P_i of an interval partition is an **interval** in the sense that if $t, t'' \in P_i$, and $t \succeq_d t' \succeq_d t''$, then $t' \in P_i$.

To summarize, if P is an equilibrium partition, then P is an interval partition with $V_h(P_i)$ increasing in i. With one technical caveat, the converse is also true.[22] As a consequence, unrefined PBE pin down very little about which sets are pooled.[23] However, focusing on the ROE guarantees additional structure on pooled sets.

[22] The remaining feature is that there must exist a *pooling strategy* for each P_i, i.e. $\sigma : P_i \to \Delta P_i$ such that each on path type declaration in P_i induces the same best response from the receiver, and off-path beliefs can be set sufficiently low. This condition is not pivotal in the analysis and so I defer its characterization to Appendix A.

[23] For an example of non-uniqueness consider that $T \equiv \{t_1, t_2, t_3\}$ with $t_3 \succeq_d t_2 \succeq_d t_1$, U^R is

4.2. The Receiver Optimal Equilibrium Partition

In the ROE, pooled sets have the additional property that they cannot be further "separated". Consider splitting some pooled set P into two parts $\underline{P}$ and $\overline{P}$, where $V_h(\overline{P}) \geq V_h(P) \geq V_h(\underline{P})$. Clearly this allocation provides the receiver with more information about the sender's type, so in the ROE this must be prevented by the sender's incentives. One possibility is that some sender types in $\underline{P}$ have the ability to mimic types in $\overline{P}$, i.e. $W(\underline{P}) \cap \overline{P} \neq \emptyset$. This is formalized below.

Definition 2. The receiver's best response, V_h, is **downward biased** on $(S, \succeq_d)$ if

$$V_h(W(\tilde{S}) \cap S) \geq V_h(S)\ \forall \tilde{S} \subset S. \tag{4}$$

I refer to sets of types over which the receiver's best response is downward biased as **downward biased sets**. The downward biased condition says that every lower contour subset of S, i.e., a subset that cannot mimic any type in its complement, has lower value than the set as a whole. Because of the assumptions on U^R, the value of each lower contour subset is also lower than its complement in S.

It is important to note that S being downward biased does not imply v is decreasing on $(S, \succeq_d)$. Figure 4 exemplifies this: v is non-monotonic in the disclosure order on the pooled set $\{t_\emptyset, A^+, A^+B^+\}$ —- $v(t_\emptyset) = .5$, $v(A^+) = .6$, and $v(A^+B^+) = 0$ — but this set is downward biased under the uniform distribution g. To see this, note that there are two proper lower contour subsets — $\{t_\emptyset\}$, and $\{t_\emptyset, A^+\}$ with $V_g(\{t_\emptyset\}) = .5$ and $V_g(\{t_\emptyset, A^+\}) = .55$ both greater than $V(\{t_\emptyset, A^+, A^+B^+\}) = .366$.

Proposition 1. *Let P be a partition of T, where $V_h(P_i)$ is increasing in i. P is the unique receiver optimal equilibrium partition if and only if*

$$V_h \text{ is downward biased on } (P_i, \succeq_d)\ \forall i, \text{ and} \tag{5}$$

$$(P_1, ..., P_m) \text{ is an interval partition of } (T, \succeq_d). \tag{6}$$

If each part of an equilibrium partition is downward biased, then it cannot be

---

quadratic loss, and $v(t_1) = 0$, $v(t_2) = 10$, and $v(t_3) = -5$. Under any distribution $h \in \Delta T$ such that $h(t_2) = h(t_3)$, the ROE partition is given by $(\{t_1\}, \{t_2, t_3\})$. However, full pooling where all types declare t_1 is also a PBE (with any other declaration assumed to come from t_3). Theorem 1 does not hold for this pooling equilibrium, as the equilibrium action for all types is given by $V_h(T) = 5/2H(\{t_2, t_3\})$ which is strictly increasing in $H(\{t_2, t_3\})$ – a more evidence change.

refined and preserve sender incentive compatibility. To see this, consider a new candidate equilibrium partition that refines some downward biased part P_i into $\tilde{P}_1, \tilde{P}_2, ..., \tilde{P}_k$ where $V_h(\tilde{P}_i)$ is strictly increasing in i; in particular $V_h(\tilde{P}_1) < V_h(P_i)$. Because of sender incentive compatibility, it must be that each $t \in \tilde{P}_1$ cannot mimic any type in $P_i \setminus \tilde{P}_1$, or alternatively, $\tilde{P}_1$ is a lower contour subset of P_i. But this yields a contradiction, as P_i being downward biased implies that $V_h(\tilde{P}_1) \geq V_h(P_i)$.

However, alternative equilibrium partitions can also be incomparable with the receiver optimal partition, i.e. neither refinements nor coarsenings. To get an intuition for the argument for Proposition 1, let P^* be an interval partition satisfying the downward biased condition on each part, and let P be some arbitrary alternative equilibrium partition. I show that the receiver does better under P^* than under P on *each* part P_i^*. This is potentially counter-intuitive because the receiver can assign types in P_i^* a variety of actions under P while pooling them all at $V_h(P_i^*)$ under P^*. It turns out that because of the downward biased property, the variety in actions under P is tailored to exactly oppose the receiver's preferences.

To see why, let $a_k \equiv V_h(P_k)$ and let $Q_k \equiv P_j \cap P_i^*$ refer to the types in P_i^* that get action a_k under P with $a_1 < ... < a_{k'} < V_h(P_i^*) < a_{k'+1} < ... < a_n$. Observe that for each j, $\cup_{k=1}^{j} Q_k$ is a lower contour subset of P_i^* and so by the downward biased property $V_h(\cup_{k=1}^{j} Q_k) \geq V_h(P_i^*)$. Similarly $\cup_{k=j}^{n} Q_k$ is an upper contour subset of P_i^* and so $V_h(\cup_{k=j}^{n} Q_k) \leq V_h(P_i^*)$. That is, P gives lower actions than P^* to subsets of types for which the receiver actually prefers higher actions and vice versa. Figure 5 illustrates this pattern: the action a_k under P is *further away* from the optimal action for that set than is $V_h(P_i^*)$. The proof of Proposition 1 transforms the alternative allocation by iteratively moving the actions down to $V_h(P_i^*)$ for the lower contour subsets (respectively up for the upper contour subsets) in a way that improves the receiver's utility at each stage.

4.3. Solving for the Receiver Optimal Equilibrium

Lemma 1. *For any set $(S, \succeq_d)$, let $\underline{J} \subset \arg\min_{\tilde{S} \subset S} V_h(W(\tilde{S}) \cap S)$ and $\overline{J} \subset \arg\max_{\tilde{S} \subset S} V_h(B(\tilde{S}) \cap S)$. Both $\cup_{\hat{S} \in \underline{J}} W(\hat{S}) \cap S$ and $\cup_{\hat{S} \in \overline{J}} B(\hat{S}) \cap S$ are downward biased sets.*

Lemma 1 says that V_h is downward biased on any minimal-valued lower contour subset or maximal valued upper contour subset. If there are multiple extrema, then V_h is downward biased on any union. The ability to find downward biased

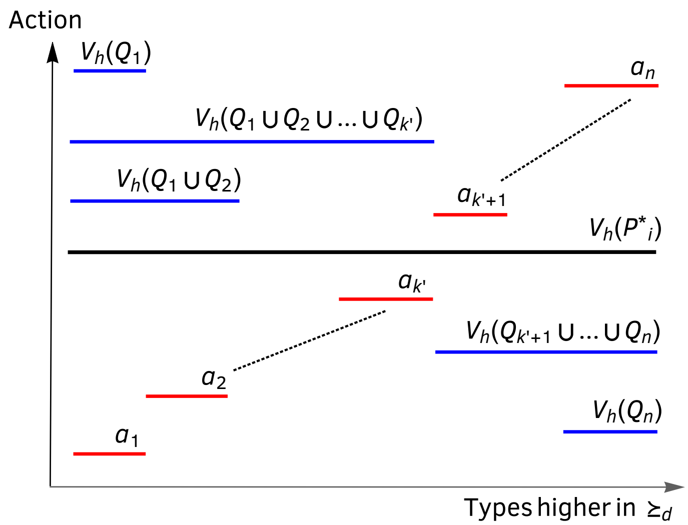


Figure 5: An ROE part P_i^* intersects an alternative partition.

sets is useful in finding the ROE partition. Consider applying the above result as follows. Begin with the entire type set T and use Lemma 1 to find a downward biased P_1. Next remove P_1 and apply Lemma 1 to $T \setminus P_1$ to find another downward biased set P_2. Repeat this process, until the type space is exhausted. This algorithm, which I call *partition into pooled sets*, generates the ROE partition.

ALGORITHM 1: Partition into Pooled Sets

Input: $(T, \succeq_d)$

Output: ROE partition

$i = 1$; $S_1 = T$;

while $S_i \neq \emptyset$ **do**

  $\overline{P}_i = \arg\min_{\tilde{S}_i \subset S_i} V_h(W(\tilde{S}_i) \cap S_i)$;

  $P_i = \cup_{S \in \overline{P}_i} S$;

  $i = i + 1$;

  $S_i = S_{i-1} \setminus P_{i-1}$;

end

Proposition 2. *The output of "Partition into Pooled Sets" is the ROE partition.*

The algorithm constructs the ROE partition "bottom-up", i.e. starting with the lowest payoff part.[24] Acharya et al. (2011) showed that, in the Dye model, the

[24] When the minimization is replaced with a maximization, and the W operator is replaced with

lowest payoff obtained by the sender is the minimum valued set that contains $t_\emptyset$; a result they termed "the minimum principle". Proposition 2 generalizes this insight: the lowest action pooled set is a minimal valued *lower contour subset*.

To illustrate the algorithm consider the example from Figure 4 with the uniform distribution g over T. Any lower contour subset contains the no evidence type $t_\emptyset$. Since A^+B^- and A^-B^+ are the highest value types and also undominated according to $\succeq_d$, they cannot be included in the minimal value lower contour subset. Similarly, A^+B^+ and A^-B^- are undominated types with the lowest values and dominate the higher value types A^+ and A^- respectively, and so if the former types are included so are their latter counterparts. This reveals that the relevant lower contour subsets for comparison and their associated values are given by $v(t_\emptyset) = .5$, $V_g(\{t_\emptyset, A^-, A^-, B^-\}) = .433$, $V_g(\{t_\emptyset, A^+, A^+B^+\} = .366$; so $\{t_\emptyset, A^+, A^+B^+\} = P_1$ is the lowest valued pooled set. Excluding these types leaves a simple Dye evidence structure composed of types $\{A^-, A^-B^+, A^-B^-\}$, and an isolated A^+B^- type. This immediately gives the equilibrium behavior illustrated in Figure 4.

While the above algorithm provides a way to determine the whole equilibrium allocation, the following theorem provides a single program that characterizes the equilibrium action for an arbitrary type.

Theorem 2. *The ROE allocation satisfies*

$$\pi_h(t|U^R) = \min_{\{S_a:\, t\in S_a\}} \max_{\{S_b:\, t\in S_b\}} V_h(W(S_a) \cap B(S_b)). \tag{7}$$

The interpretation of Theorem 2 is that the pooled set for a given type t results from the combination of two forces. First, type t chooses some set of dominated types to pool with — $B(S_b)$ — in order to increase the receiver's best response. Second, the types dominating this chosen set — $W(S_a)$ — will pool with t if it improves their value. This latter process serves to lower the value as these more dominant types will only pool with t if they have relatively lower value. Thus the min-max in (7) comes from (i) types that dominate t pooling with t to *minimize* his action (because it improves their own) and (ii) t pooling with types he dominates in order to *maximize* his action. The program in (7) reveals the complexity in the

the B operator, the algorithm constructs the same equilibrium partition "top-down", i.e. starting from the highest payoff part.

general disclosure game: a type t considering mimicking t' cannot just consider the value of t' but also all the types in $B(t')$, i.e., those that also have the ability to mimic t', and of course, this consideration also flows in the opposite direction when types consider mimicking t.

Application to Honest Types Theorem 2 can be applied to the honest types model from Subsection 2.1. Recall that the type space $T \equiv T' \times \{S, H\}$ is composed of disclosable evidence $\{T', \succeq'_d\}$ and an indicator $\theta \in \{S, H\}$ for whether the sender is strategic S and can disclose according to $\succeq'_d$ or honest H and must truthfully reveal $t \in T'$. For any $R \subset T'$, define

$$\tilde{V}(R) \equiv \max_{\tilde{H} \subset R} V_h((R \times \{S\}) \cup (\tilde{H} \times \{H\})). \tag{8}$$

This is the receiver's best response to the strategic senders with evidence in R and the honest senders with evidence $\tilde{H} \subset R$. The selected $\tilde{H}$ is the subset of types in R who have higher value than $\tilde{V}(R)$ and so mechanically $\tilde{V}(R) \geq V_h(R),\ \forall R \subset T'$.

Proposition 3. *The ROE allocation in an honest types model is given by*

$$\begin{aligned} \pi_h((t,S)|U^R) &= \min_{\{S_a \subset T':\, t \in S_a\}} \max_{\{S_b \subset T':\, t\, \in S_b\}} \tilde{V}(W(S_a) \cap B(S_b)), \\ \pi_h((t,H)|U^R) &= \min\{\pi_h((t,S)|U^R), v(t)\}. \end{aligned} \tag{9}$$

The ROE actions for strategic types are the same as in a standard disclosure game without the honest senders, but where the receiver has more favorable beliefs about the sender: his best response to all subsets shifts up from V_h to $\tilde{V}$. On the other hand the receiver obtains his bliss point for any nonstrategic sender with value less than the ROE action of his strategic counterpart. The argument for the result uses Theorem 2. The idea is that S_b in (7) can be altered to include or exclude arbitrary honest counterparts of the strategic types in $W(S_a) \cap B(S_b)$. Since S_b is chosen to maximize the receiver's value, this yields the objective in (8).

5. Why More Evidence implies More Skepticism

The previous section characterized pooled sets as those over which the receiver's best response is downward biased. Still, because the downward biased property does not imply monotonicity, we cannot employ standard comparative statics to show that the value of a pooled set decreases under a more evidence shift. Establishing this fact, which serves as the backbone for the proof of Theorem 1, is the goal of this section. For ease of exposition, I assume all distributions have full support and that the receiver has quadratic loss so that $V_f(S) = \mathbb{E}[v(s)|s \in S, s \sim f]$.[25]

Proposition 4. *Let* $(S, \succeq_d)$ *and* $f \in \Delta S$. $V_f(S) \leq V_g(S)$ $\forall g \in \Delta S$ *such that* $f \geq_{ME} g$ *with respect to* $(S, \succeq_d)$ *if and only if* V_f *is downward biased on* $(S, \succeq_d)$.

The result says that the condition that characterizes pooled sets in the ROE also characterizes monotone comparative statics (MCS) under a likelihood ratio increase in the distribution up a partially ordered set, i.e., a more evidence shift. Proposition 4 implies that equilibrium actions are lower under more evidence changes in the receiver's beliefs that preserve the ROE partition. At the end of this section I discuss how to adapt this conclusion to the case in which the ROE partition changes under a more evidence shift.

One direction of Proposition 4 is relatively straightforward. If V_h is not downward biased on $(S, \succeq_d)$, there is a lower contour subset with lower value than S as a whole. Moving probability from this subset to its complement is a more evidence shift and increases the value of S. The other direction in Proposition 4 is more complicated. One would like to use the following well known comparative statics result: the expected value of a decreasing function is lower under a monotone likelihood ratio shift. This result appeared in Topkis (1976) and is formally reproduced below in terms of receiver best responses.

Fact 1. *For* $f, g \in \Delta S$, *if* $\forall t, t' \in S$, $f(t)g(t') \geq f(t')g(t) \implies v(t) \leq v(t')$, *then* $V_f(S) \leq V_g(S)$.

If the disclosure order were complete, and the receiver's best response were decreasing in the disclosure order on any pooled set, the above fact would yield Proposition 4. The problem is that the disclosure order is not complete, and even if it were, as Figure 4 illustrates, no such monotonicity property holds on the ROE pooled sets. The next section instead uses the above fact iteratively to establish MCS for downward biased sets.

[25] Subsection B.7 shows how to use these results to prove Theorem 1 which does not assume $f, g \in \Delta T$ have full support and the receiver can have arbitrary utility $U \in \Upsilon^R$.

5.1. Iteratively Pooling Subsets

Consider a downward biased set $(S, \succeq_d)$ and two distributions $f, g \in \Delta S$ such that $f \geq_{ME} g$. For ease of exposition, assume that for any two different types $t', t'' \in S$, $\frac{f(t')}{g(t')} \neq \frac{f(t'')}{g(t'')}$. Order types according to their likelihood ratio as $S = \{t_1, ..., t_n\}$ where $\frac{f(t_i)}{g(t_i} > \frac{f(t_j)}{g(t_j)} \iff i > j$. Because $f \geq_{ME} g$, this likelihood ratio order refines the disclosure order $\succeq_d$, i.e., $t_i \succeq_d t_j \implies i \geq j$. The important implication is that any lower truncated set of types, $\{t_1, ..., t_l\}$, is also a lower contour subset of S. This means that the downward biased property implies that $V_f(\{t_1, ..., t_l\}) \geq V_f(S)\ \forall l = 1, ..., n$. I prove Proposition 4 by appealing to the algorithm below.

Description of the Algorithm The algorithm begins with the complete partition, $Q^1 = (\{t_1\}, \{t_2\}, ..., \{t_n\})$. Beginning with t_1, the algorithm repeatedly forms the largest consecutive sequence of elements such that $v(t_j)$ is decreasing in j. That is, the first sequence is $\{t_1, t_2, ..., t_{I_1}\}$ such that $v(t_1) \geq ... \geq v(t_{I_1})$ and $v(t_{I_1}) < v(t_{I_1+1})$, the second sequence is $\{t_{I_1+1}, ..., t_{I_2}\}$ such that $v(t_{I_1+1}) \geq ... \geq v(t_{I_2})$ and $v(t_{I_2}) < v(t_{I_2+1})$, and so on until all types in S are exhausted. Next, a coarser partition Q^2 is formed by pooling all the elements of each decreasing sequence into an associated single part with value determined by $V_f(\cdot)$. That is, $Q_1^2 \equiv \{t_1, ..., t_{I_1}\}$, $Q_2^2 = \{t_{I_1+1}, ..., t_{I_2}\}$, and so on. This process is repeated: at each stage, Q^i is coarsened into Q^{i+1} where each part of Q^{i+1} pools a consecutive sequence of Q_j^i over which $V_f(Q_j^i)$ is decreasing in j. The algorithm concludes at stage T defined by $Q^T = Q^{T+1}$, i.e. where $V_f(Q_i^T)$ is strictly increasing in i.

Proof Sketch of Proposition 4 Consider an arbitrary partition element, Q_i^t, at stage $t > 1$. Because each stage coarsens the partition, Q_j^i is composed of "adjacent" parts from the previous partition, i.e., $Q_j^i = \cup_{l=\underline{l}}^{\bar{l}} Q_l^{i-1}$ for some $\underline{l} < \bar{l}$, where $V_f(Q_l^{i-1})$ is decreasing in l for $l = \underline{l}, \underline{l}+1, ..., \bar{l}$. This means that, using Fact 1,

$$\sum_{l=\underline{l}}^{\bar{l}} V_f(Q_l^{i-1}) G(Q_l^{i-1}) \geq \frac{G(Q_j^i)}{F(Q_j^i)} \sum_{l=\underline{l}}^{\bar{l}} V_f(Q_l^{i-1}) F(Q_l^{i-1}) = G(Q_j^i) V_f(Q_j^i),$$

where the last line makes use of the fact that the receiver has quadratic loss so that V_f is a conditional expectation. By summing over all parts in Q^i, we get

$$\sum_l V_f(Q_l^{i-1}) G(Q_l^{i-1}) \geq \sum_l V_f(Q_l^i) G(Q_l^i).$$

Since this inequality holds at each stage, it holds between $i = 1$, where each part is a single element, and the terminal stage $i = T$, i.e.,

$$V_g(S) = \sum_i v(t_j)g(t_i) \geq \sum_i V_f(Q_i^T)G(Q_i^T).$$

If the terminal partition is trivial, i.e., $Q_1^T = S$, then the right hand side is $V_f(S)$, which completes the argument. If the terminal partition is not trivial, then $V_f(Q_1^T) < V_f(S)$ which contradicts the downward biased property.

Illustration Recall the pooled set $P_1 = \{t_\emptyset, A^+, A^+B^+\}$ from Figure 4 which is downward biased under the uniform distribution g. Let $f \geq_{ME} g$, which in this case means that f MLR dominates g on the ordered set $(t_\emptyset, A^+, A^+B^+)$.

The algorithm starts at the complete partition — $Q^1 = (\{t_\emptyset\}, \{A^+\}, \{A^+B^+\})$, and forms maximum decreasing sequences according to v and the disclosure order. These sequences are collapsed into new elements of the partition Q^2, i.e., $Q_1^2 = \{t_\emptyset\}$ and $Q_2^2 = \{A^+, A^+B^+\}$. Notice that because the receiver's value is decreasing on each part of this new partition, $V_g(Q_i^2) \geq V_f(Q_i^2)\ \forall i = 1, 2$.

The process repeats and, because $V_g(Q_1^2) = v(\{t_\emptyset\}) = .5 \geq .3 = V_g(\{A^+, A^+B^+\}) = V_g(Q_2^2)$, forms the sequence (Q_1^2, Q_2^2) coarsening Q^2 into the trivial partition. Notice again that because $V_g(Q_1^2) \geq V_g(Q_2^2)$,

$$V_g(Q_1^2)F(Q_1^2) + V_g(Q_2^2)F(Q_2^2) \leq V_g(Q_1^2)G(Q_1^2) + V_g(Q_2^2)G(Q_2^2).$$

Putting the inequalities gleaned at each stage together gives the desired conclusion that $V_f(P_1) \leq V_g(P_1)$.

5.2. Changes in the Equilibrium Partition

Proposition 4 is only sufficient to show that more evidence implies lower equilibrium actions in the case when the ROE partition is constant. This subsection provides intuition for extending the argument to cases in which the ROE partition changes when the sender has more evidence.

Consider that $f \geq_{ME} g$. For simplicity, let the associated ROE partitions $P^g = (P_1^g, P_2^g)$ have two elements, while $P^f = (P_1^f)$, has only one. For $\alpha \in [0, 1]$, define the combination distribution, $h_\alpha \equiv \alpha f + (1-\alpha)g$ with corresponding ROE partition

$P^\alpha \equiv (P_1^\alpha, ..., P_{m_\alpha}^\alpha)$. By Proposition 1 $V_g(P_1^g) < V_g(P_2^g)$, and $V_f(P_1^g) \geq V_f(P_2^g)$.

Because the receiver's best response to these subsets is continuous in α, there must exist some α^* above which the ROE partition changes from P^g to P^f and $V_{h_{\alpha^*}}(P_1^g) = V_{h_{\alpha^*}}(P_2^g)$. For simplicity, suppose that this is the only change in the ROE partition as α increases from 0 to 1. Notice that for $\alpha > \alpha'$, $h_\alpha \geq_{ME} h_{\alpha'}$, and so Proposition 4 completes the argument in this case. The idea is that each equilibrium part – P_1^g and P_2^g – decreases in value as α increases, until equalizing at $\alpha = \alpha^*$. For $\alpha > \alpha^*$, all types pool together and so again by Proposition 4, the value of $P_f^1 = S$ decreases in α.

6. Application to Dynamic Disclosure

Consider an entrepreneur making progress on a project that he can eventually disclose to a venture capitalist in order to obtain an investment. Naturally this progress happens gradually: first, perhaps the entrepreneur attempts to develop a prototype, and only after can he potentially run performance test. This presents the investor with the opportunity to speak with the entrepreneur at some intermediate stage when potentially not all the evidence has arrived. Can the investor benefit from these additional communications or should he just wait until making his investment to consult with the entrepreneur? The depth in this question is particular to multidimensional evidence structures: if the entrepreneur could potentially obtain only a single piece of evidence as in the Dye model, any early signaling would essentially end the game.

Broad lessons from dynamic mechanism design speak to the potential benefits of this early communication when the entrepreneur has less private information about his eventual progress. On the other hand, in a framework where investment decisions are made only based on final disclosures, one might doubt whether early disclosures would ever be made by the entrepreneur: why would he want to disclose early progress only to set expectations high for the future? Below, I develop a simple two period extension to the static disclosure model, and characterize when the receiver can and cannot benefit from early disclosure.

6.1. Dynamic Arrival of Evidence

A single sender and a single receiver interact over two periods. The disclosure in each period follows the structure of the main text model in Section 2. The sender has evidence $t_i \in T$ in period i, where $|T| = n$. In each period i, the sender sends a message $s_i \in T$ which is constrained according to the disclosure order $\succeq_d$, i.e., $s_i \in \{s : \; t_i \succeq_d s\}$. I assume that there is a "no evidence type" $t_\emptyset \in T$ such that $t \succeq_d t_\emptyset \; \forall t \in T$. Sending $t_\emptyset$ in either period is interpreted as non-disclosure. The type space and disclosure order, $(T, \succeq_d)$, remain the same across periods. After observing the disclosure in each period, s_1 and s_2, the receiver takes an action a to maximize $U^R(a, t_2)$ where $U^R \in \Upsilon$. That is, the receiver only cares about the final evidence of the sender when selecting his action. I maintain that the sender simply wants to maximize the chosen action a.

The sender's evidence arrives gradually, and so his type changes between periods. The distribution of evidence t_1 in period 1 is given by $h_1 \in \Delta T$ which is assumed to have full support. The probability of evidence t_2 in period 2, conditional on t_1 in period 1, is given by

$$h_2(t_2|t_1) = \begin{cases} \frac{\tilde{h}(t_2)}{\tilde{H}(B(t_1))} & \text{if } t_2 \in B(t_1) \\ 0 & \text{otherwise} \end{cases},$$

for some $\tilde{h} \in \Delta T$ with full support. This implies that possessing more evidence in period 1 makes one more likely to have more evidence in period 2 in the sense of Definition 1. Note that for $t_2 \not\succeq_d t_1$, the probability of realizing t_2 after t_1 is zero, i.e. the sender does not "lose evidence" over time, and, for simplicity, this is the only way that t_2 depends on t_1. Also, evidence is not "time-stamped", i.e., the sender cannot credibly convey in period 2 when any disclosed evidence arrived.

I focus on PBE of this game with the additional assumption that the ROE characterized in Theorem 2 is selected in period 2. That is, denote the receiver's interim beliefs about t_2 — after observing s_1, but before observing s_2 — $h_{s_1} \in \Delta T$. The equilibrium allocation as a function of the type is determined according to $\pi_{h_{s_1}}(t_2|U^R) \; \forall t_2 \in \text{Supp}(h_{s_1})$.[26,27]

[26] Formally, given sender period 1 disclosure strategies $\sigma_1 : T \to \Delta T$, if $s_1 \in \cup_{t_1 \in T} \text{Supp}(\sigma_1(t_1))$, let $\tilde{h}_1(\cdot) \equiv \mathbb{P}(\cdot|s_1)$ be the receiver's beliefs about t_1 computed according to Bayes Rule, and otherwise let $\tilde{h}_1 \in \Delta(B(s_1))$. The interim belief given disclosure s_1 is $h_{s_1}(t_2) \equiv \sum_{t_1 \in T} h_2(t_2|t_1)\tilde{h}_1(t_1)$. Given this refinement, equilibria are described by a sender period 1 strategy $\sigma_1 : T \to \Delta T$, and interim beliefs $h : T \to \Delta T$, such that $\text{Supp}(\sigma_1(t_1)) \subseteq \arg\max_{s_1 \in W(t_1)} \mathbb{E}[\pi_{\tilde{h}_{s_1}}(t_2|U^R)|t_2 \sim h_2(\cdot|t_1)]$.

[27] Recall that this equilibrium focus also corresponds to the receiver having commitment power within but not across periods, or to the truth leaning refinement a la Hart et al. (2017) in period 2.

For a given equilibrium of this dynamic disclosure game, denote $\tilde{\pi} : T \to \Delta\mathbb{A}$ as the distribution of equilibrium actions as a function of the sender's period 2 type.[28] Let the ex-ante distribution over period 2 types be denoted by $h_A \in \Delta T$.[29] One equilibrium, which I term **static communication**, involves the sender disclosing $t_\emptyset$ in period 1 regardless of t_1. This induces an allocation that is degenerate on $\pi_{h_A}(t_2|U^R)$ for every $t_2 \in T$, i.e., equivalent to the ROE in a static disclosure model. I say the receiver **benefits from early inspections** if there exists an equilibrium allocation $\tilde{\pi}$ which the receiver prefers to the static communication equilibrium. More specifically, the receiver benefits from early inspections if there is an equilibrium $\tilde{\pi}$ such that $\tilde{\pi}(t_2)$ is non-degenerate for some $t_2 \in T$. Such non-degeneracy represents the receiver obtaining instrumental information in period 1. Thus, the question of whether the receiver benefits from early inspections reduces to whether there exists an equilibrium in which the sender makes informative disclosures in period 1. The following feature of disclosure orders is pivotal to this question.

Definition 3. A disclosure-ordered type space $(T, \succeq_d)$ has the unique evidence path property (UEPP) if $\forall t, t', t'' \in T$, $t \succeq_d t'$ and $t \succeq_d t'' \implies t' \succeq_d t''$ or $t'' \succeq_d t'$.

An alternative description of the UEPP is that for any type t, $W(t)$ is completely ordered. In this sense, the UEPP says there is a unique "path" in the disclosure order to each type. While every Dye model satisfies the UEPP, this is not true for multidimensional evidence structures: the UEPP is satisfied in the sequential evidence collection model in the right panel of Figure 2, but not in the independent evidence collection framework in the left panel of that figure.[30] Indeed, the main interpretation of the UEPP is that the evidence is gathered through a sequential process of investigations uniquely determined by the results of the previous one. The motivating example of this section satisfies the UEPP: a prototype can only be tested if it has first been successfully developed, and so revealing a successful performance test would also reveal which prototype was developed. In the context of criminal investigations, an alibi can only be reported if the suspect has first been identified. If instead different pieces of evidence can be collected independently, then the UEPP will not be satisfied.

[28] The receiver does not randomize over actions in period 2, rather the potential randomness in $\tilde{\pi}$ arises due to different period 1 disclosures leading to the same period 2 disclosure.

[29] That is, $h_A(t) = \sum_{s \in T} h_1(s) h_2(t|s)$.

[30] Note that $\{0, 1\} \succeq_d \{0\}$ and $\{0, 1\} \succeq_d \{1\}$, but $\{1\}$ and $\{0\}$ are not ordered according to $\succeq_d$.

Proposition 5. *If* $(T, \succeq_d)$ *satisfies the UEPP, then the receiver does not benefit from early inspections. If* $(T, \succeq_d)$ *does not satisfy the UEPP, then there exists* h_1, h_2*, and* U^R *such that the receiver benefits from early inspections.*

The more complicated implication is that the receiver does not benefit from early inspections under the UEPP, i.e., that all equilibria are outcome equivalent to static communication. Recall that if instrumental period 1 disclosures could be elicited, those that lead to different ROE actions in period 2, they would be valuable even under the UEPP. The result holds because such early disclosures are not incentive compatible for the sender. The key intuition is that disclosing evidence in period 1 is bad for the sender because it effectively tells the receiver that "more evidence" is expected in period 2 as compared to if such evidence was not held in period 1. This in turn induces worse equilibrium actions from the receiver because of Theorem 1. Without the UEPP two available disclosures for a given sender type t may be incomparable according to $\succeq_d$, and so one disclosure may not signal "more evidence" in the eyes of the receiver.[31]

Proposition 5 suggests that if the sender is deciding only "how much" evidence to disclose, as in the sequential collection evidence structure, then the receiver cannot benefit from the sender's evidence arriving gradually. Alternatively, under the independent collection case, where the sender additionally decides "which" evidence to disclose, the receiver can benefit by speaking to the sender before he has acquired all evidence. Importantly, such a distinction does not appear in the Dye evidence model, and so this takeaway could not be gleaned with existing comparative statics results.

A. Preliminaries

Lemma 2. *Consider two distributions* $q_1, q_2 \in \Delta T$ *such that* $V_{q_1}(T) < (=) V_{q_2}(T)$*. For any* $\lambda \in (0,1)$*,* $V_{q_1}(T) < (=) V_{\lambda q_1 + (1-\lambda) q_2}(T) < (=) V_{q_2}(T)$*.*

Proof. This follows directly from Hart et al. (2017) lemma 1. *Q.E.D.*

A set $(S, \succeq_d)$ is **poolable** with respect to full support $h \in \Delta S$ if there exists a feasible sender strategy and receiver best responses on path $a^\sigma : \cup_{s \in S} \operatorname{Supp}(\sigma_s) \rightarrow$

[31] In my earlier working paper Rappoport (2020), Appendix F provides an example with informative dynamic signaling when the UEPP is violated.

A such that $a^\sigma(s) = V_h(S)\ \forall s \in \cup_{s\in S} \operatorname{Supp}(\sigma_s)$. Define the following useful notation for a partially ordered set $(S, \succeq_d)$:

$$
\begin{aligned}
&(i)\ \underline{W}(S) \equiv \{t \in S :\ \forall s \in S\ t \not\succ_d s\},\\
&(ii)\ V^+(S) \equiv \{s \in S :\ v(s) \geq V_h(S)\},\\
&(iii)\ \forall W' \subset \underline{W}(S)\ E(W') \equiv B(W') \setminus B(\underline{W}(S) \setminus W'),\ \textit{and}\\
&(iv)\ \forall W' \subset \underline{W}(S)\ Q(W') \equiv E(W') \cup (B(W') \cap V^+(S)).
\end{aligned}
$$

Lemma 3. *a partially ordered set.$(S, \succeq_d)$ is poolable with respect to $h \in \Delta S$ if and only if*

$$\forall W' \subset \underline{W}(S) \quad V_h(Q(W')) \geq V_h(S). \tag{10}$$

Proof. " $\implies$ " Take a pooling strategy σ. Note that it is without loss to take $\operatorname{Supp}(\sigma) \subset \underline{W}(S)$.[32] Take $W' \subset \underline{W}(S)$. Since σ is pooling, the best response must be $a^\sigma(w) = V_h(S)\ \forall w \in \operatorname{Supp}(\sigma)$. Let

$$q(t) \equiv \frac{\sum_{s\in W'} \sigma_t(s)h(t)}{\sum_{s'\in S}\sum_{s\in W'} \sigma_{s'}(s)h(t)},$$

and note that by Lemma 2 and the fact that σ is pooling, $V_q(S) = V_h(S)$. By definition, $q(t) = h(t)\ \forall t \in E(W')$. Moreover, the induced distribution of types given $Q(W')$ adds probability of types with value greater than $V_h(S)$ and decreases the probability of types with value less than $V_h(S)$, by Lemma 2 it must be that $V_h(S) \leq V_h(Q(W'))$.

" $\impliedby$ " Let $\mathcal{H} \equiv \{g \in \Delta S :\ \underline{W}(\operatorname{Supp}(g)) \subset \underline{W}(S)\}$. By definition $h \in \mathcal{H}$. I show that there exists a pooling strategy on $(S, \succeq_d)$with respect to every $g \in \mathcal{H}$ that satisfies the assumptions of the proposition by induction on $|\underline{W}(\operatorname{Supp}(g))|$. For the base case of $\underline{W}(\operatorname{Supp}(g)) = \{w\}$, the strategy $\sigma_t(w) = 1\ \forall t \in S$ is a pooling strategy.

Now let there exist pooling strategies for all $g \in \mathcal{H}$ that satisfy (10) on $\underline{W}(\operatorname{Supp}(g))$, such that $|\underline{W}(\operatorname{Supp}(g))| = N$ and consider a distribution $g' \in \Delta S$ with $|\underline{W}(\operatorname{Supp}(g'))| = N + 1$ that satisfies (10) on $\underline{W}(\operatorname{Supp}(g')) \equiv W$. Consider arbitrary $w \in W$. First consider the case in which $V_{g'}(E(\{w\})) \leq V_{g'}(S)$. For $\lambda \in [0, 1]$ define C_λ and

[32] For each $t \in \cup_{s\in S} \operatorname{Supp}(\sigma_s) \setminus \underline{W}(S)$, identify arbitrary $\underline{t}(t) \in \underline{W}(S)$. For each $w \in \underline{W}(S)$ define $\tilde{\sigma}_s(w) = \sum_{\tilde{t}: w=\underline{t}(\tilde{t})} \sigma_s(\tilde{t})$ and $\tilde{\sigma}_s(t) = 0$ otherwise.

distribution $f_\lambda \in \Delta S$ as follows

$$C_\lambda \equiv \sum_{s \in S} \left(\lambda \mathbb{1}_{s \in Q(\{w\})} + (1-\lambda)\mathbb{1}_{s \in E(\{w\})}\right) g'(s),$$
$$f_\lambda(s) \equiv \left(\lambda \mathbb{1}_{s \in Q(\{w\})} + (1-\lambda)\mathbb{1}_{s \in E(\{w\})}\right) g'(s)/C_\lambda \ \forall s \in S.$$

First, because $E(\{w\}) \subset Q(\{w\})$ $f_\lambda(s) = g'(s)/C_\lambda$ $\forall s \in E(\{w\})$, i.e. f_λ includes all probability mass of types that must declare w. Second, note that $f_1 = g'_{|Q(\{w\})}$ so $V_{f_1}(S) = V_{g'}(Q(\{w\}))$ which means that $V_{f_1}(S) \geq V_{g'}(S)$ by (10). Third, note that $f_0 = g'_{|E(\{w\})}$ so $V_{f_0}(S) = V_{g'}(E(\{w\}))$ which means that $V_{f_0}(S) \leq V_{g'}(S)$ by assumption. Since $V_{f_\lambda}(S)$ is continuous in λ, $\exists \lambda \in [0,1]$ such that $V_{f_\lambda}(S) = V_{g'}(S)$.

Suppose now that $V_{g'}(E(\{w\})) \geq V_{g'}(S)$. By definition $Q(W \setminus \{w\}) \cap E(\{w\}) = \emptyset$ and $V_{g'}(Q(W \setminus \{w\})) \geq V_{g'}(S)$ by (10). Let $R \equiv Q(W \setminus \{w\})^c$. This means that $E(\{w\}) \subset R$ and $V_{g'}(R) \leq V_{g'}(S)$ by Lemma 2. Now for $\lambda \in [0,1]$ redefine

$$C_\lambda \equiv \sum_{s \in S} \left(\lambda \mathbb{1}_{s \in R} + (1-\lambda)\mathbb{1}_{s \in E(\{w\})}\right) g'(s),$$
$$f_\lambda(s) \equiv \left(\lambda \mathbb{1}_{s \in R} + (1-\lambda)\mathbb{1}_{s \in E(\{w\})}\right) g'(s)/C_\lambda \ \forall s \in S.$$

By symmetric logic to the above paragraph $f_\lambda(s) = g'(s)/C_\lambda$ $\forall s \in E(\{w\})$ and $V_{f_1}(S) \leq V_{g'}(S) \leq V_{f_0}(S)$. Since the receiver's best response is continuous in λ, $\exists \lambda \in [0,1]$ such that $V_{f_\lambda}(S) = V_{g'}(S)$.

Now consider the distribution, $g'' \in \Delta S$ defined by $g''(s) \equiv \dfrac{g'(s) - f_\lambda(s) C_\lambda}{1 - C_\lambda}$. Since g' is a convex combination of g'' and f_λ and $V_{f_\lambda}(S) = V_{g'}(S)$, by Lemma 2 $V_{g''}(S) = V_{g'}(S)$. By definition of f_λ, $g''(s) = 0$ $\forall s \in E(\{w\})$, so $\underline{W}(\mathrm{Supp}(g'')) = \underline{W}(S) \setminus \{w\}$. Thus $|\underline{W}(\mathrm{Supp}(g''))| = N$.

Now I verify that S with distribution g'' satisfies (10). Consider arbitrary $W' \subset \underline{W}(\mathrm{Supp}(g'')) = W \setminus \{w\}$. By construction $\mathrm{Supp}(f_\lambda) \subset Q(\{w\}) \subset Q(W' \cup \{w\})$. Thus $V_{f_\lambda}(Q(W' \cup \{w\})) = V_{f_\lambda}(S) = V_{g'}(S)$. By (10) on S with respect to g', $V_{g'}(Q(W' \cup \{w\})) \geq V_{g'}(S)$. Finally, since g' is a convex combination of g'' and f_λ, by Lemma 2 $V_{g''}(Q(W' \cup \{w\})) = V_{g''}(Q(W')) \geq V_{g'}(S) = V_{g''}(S)$ and (10) is satisfied on S with respect to g''.

By the induction hypothesis, $\exists \sigma : \mathrm{Supp}(g'') \to \Delta(\underline{W}(\mathrm{Supp}(g'')))$ that is pooling with respect to g''. Now $\forall w' \in W$ define

$$\tilde{\sigma}_{t'}(w') \equiv \begin{cases} (1 - C_\lambda f_\lambda(t'))\sigma_{t'}(w') & \text{if } w' \in \underline{W}(\text{Supp}(g'')) \\ C_\lambda f_\lambda(t') & \text{if } w' = w \end{cases}.$$

The best responses to $\tilde{\sigma}$ are the best responses to σ on $W(\text{Supp}(g''))$ and the best response to f_λ on w. Thus $\tilde{\sigma}$ is a pooling strategy for g' on S. *Q.E.D.*

Lemma 4. *If V_h is downward biased on $(S, \succeq_d)$ then $(S, \succeq_d)$ is poolable with respect to h.*

Proof. Take $W' \subset \underline{W}(S)$. Because $E(W')$ is a lower contour subset of S and V_h is downward biased on S, $V_h(E(W')) \geq V_h(S)$. By Lemma 2 $V_h(Q(W')) \geq V_h(S)$. The result follows from Lemma 3. *Q.E.D.*

B. Omitted Proofs

B.1. Proof of Proposition 1

Proof. The argument for Proposition 2 provides existence of an interval partition P^* such that V_h is downward biased on P_i^* $\forall i = 1, ..., m$. The existence of pooling strategies on each P_i^* is provided by Lemma 4.[33] I will show that such a partition is the ROE partition. Suppose that $\pi : T \to \mathbb{R}$ be some alternative allocation such that $t \succeq_d t' \implies \pi(t) \geq \pi(t')$. Let $P = (P_1, ..., P_m)$ represent the associated interval partition into ordered equivalence classes induced by π.

Claim 1. *The receiver's utility is higher under P^* than under π.*

Proof of Claim: I show that the receiver's utility is higher on each part P_i^*. Let $Q_j \equiv P_l \cap P_i^*$ and $a_j \equiv \pi_l$ where l is the j'th highest index such that $P_l \cap P_i^* \neq \emptyset$ with $P_i^* = \cup_{j=1}^{\overline{k}} Q_j$. Take $\hat{k}$ such that $a_1 < ... < a_{\hat{k}} \leq V_h(P_i^*) \leq a_{\hat{k}+1} < ... < a_{\overline{k}}$. Note that because P is an interval partition, $\cup_{j=1}^{k} Q_j \equiv \underline{Q}_k$ is a lower contour subset of P_i^* for every k. This means that $V_h(\underline{Q}_k) \geq V_h(P_i^*)$ $\forall k$ because V_h is downward biased on each P_i^*. By strict concavity of U^R, moving the action closer to a set's bliss point

[33] For any off path $t' \in P_i^*$ in such a pooling strategy one can set $a(t) = V_h(B(t) \cap S) \leq V_h(P_i^*)$ where the inequality follows for the fact that V_h is downward biased on P_i^*.

for that set increases the receiver's utility. Thus $\forall k \leq \hat{k}$, $\forall\, \tilde{a} \in [a_k, V_h(P_i^*)]$.

$$\sum_{t \in \underline{Q}_k} U^R(a_k, t)h(t) \leq \sum_{t \in \underline{Q}_k} U^R(\tilde{a}, t)h(t) \leq \sum_{t \in \underline{Q}_k} U^R(V_h(P_i^*), t)h(t). \tag{11}$$

Using these inequalities in (11) gives

$$\sum_{t \in \underline{Q}_k} U^R(a_k, t)h(t) + \sum_{j=k+1}^{\hat{k}} \sum_{t \in Q_j} U^R(a_j, t)h(t) \leq \sum_{t \in \underline{Q}_{k+1}} U^R(a_{k+1}, t)h(t) + \sum_{j=k+2}^{\hat{k}} \sum_{t \in Q_j} U^R(a_j, t)h(t)$$

A sequence of these inequalities as k ranges from 1 to $\hat{k} - 1$ gives that

$$\sum_{j=1}^{\hat{k}} \sum_{t \in Q_j} U^R(a_j, t)h(t) \leq \sum_{t \in \underline{Q}_{\hat{k}}} U^R(a_{\hat{k}}, t)h(t) \leq \sum_{t \in \underline{Q}_{\hat{k}}} U^R(V_h(P_i^*), t)h(t).$$

This shows that the receiver does better on $Q_1 \cup ... \cup Q_{\hat{k}}$ under P^* than under P. A symmetric argument for higher actions shows that the receiver also does better on each upper contour subset $\cup_{j=\hat{k}+1}^{\overline{k}} Q_j$. *Q.E.D.*

B.2. Proof of Lemma 1

Proof. Let $S^* \in \arg\min_{\tilde{S} \subset S} V_h(W(\tilde{S}) \cap S)$ with $\overline{W} \equiv W(S^*) \cap S$ and $\overline{V} \equiv V_h(\overline{W})$. Suppose that $\exists \tilde{S} \subset \overline{W}$: $V_h(W(\tilde{S}) \cap \overline{W}) < V_h(\overline{W})$. Since being a lower contour subset is preserved under intersections $W(W(\tilde{S}) \cap \overline{W}) = W(\tilde{S}) \cap \overline{W}$, which contradicts the minimality of $\overline{W}$ in the above problem. Thus each minimizer of the above problem is downward biased.

Now take $J \subset \arg\min_{\tilde{S} \subset S} V_h(W(\tilde{S}) \cap S)$ with $J \equiv (S_1, ..., S_k)$, $\overline{W}_i \equiv W(S_i) \cap S$, and $\overline{W} \equiv \cup_{i=1}^k \overline{W}_i$. Note that because each $\overline{W}_i$ is downward biased and $\overline{W}_i \backslash \cup_{j=1}^{i-1} \overline{W}_j$ is an upper contour subset of $\overline{W}_i$, $V_h(\overline{W}_i \setminus \cup_{j=1}^{i-1} \overline{W}_j) \leq V_h(\overline{W}_i) = \overline{V}$. Since $\overline{W}$ is the disjoint union of these sets, i.e. $\overline{W} = \cup_{i=1}^k (W_i \setminus \cup_{j=1}^{i-1} \overline{W}_j)$, Lemma 2 implies that $V_h(\overline{W}) \leq \overline{V}$. Thus $\overline{W} \in \arg\min_{\tilde{S} \subset S} V_h(W(\tilde{S}) \cap S)$, and so by the previous argument $\overline{W}$ is downward biased. The argument is symmetric for the $\arg\max$ case. *Q.E.D.*

B.3. Proof of Proposition 2

Proof. Algorithm 1 produces a partition of T into disjoint sets $(P_1, P_2, ..., P_m)$. I argue that this partition satisfies the requirements of Proposition 1, and thereby constitutes the ROE partition. Lemma 1 implies that each P_i is a downward biased set. By construction, $V_h(P_i) < V_h(P_{i+1})$, otherwise $V_h(P_i \cup P_{i+1}) \leq V_h(P_i)$ by Lemma 2 contradicting that P_i is selected. Now suppose that $t \succeq_d t'$ with $t \in P_i$, and $t' \in P_j$ such that $i < j$. Then $W(P_i) \cap (T \setminus (\cup_{k=1}^{i-1} P_i)) \neq P_i$ again contradicting that P_i was selected. The argument is symmetric for the $\arg\max$ case. *Q.E.D.*

B.4. Proof of Theorem 2

Proof. Take the ROE partition $(P_1, ..., P_m)$. For $t \in P_i$, $V_h(P_i) = \pi_h(t|U^R)$. Thus, I prove that the solution to the problem on the right hand side of (7) is $V_h(P_i)$. Define $S_a^* \equiv \cup_{k=1}^{i} P_k$ and $S_b^* \equiv \cup_{k=i}^{m} P_k$. I show that choosing $S_a = S_a^*$ bounds the value of (7) to be less than $V_h(P_i)$. If we instead consider

$$\max_{\{S_b : t \in S_b\}} \min_{\{S_a : t \in S_a\}} V_h(W(S_a) \cap B(S_b)). \tag{12}$$

The argument for why choosing $S_b = S_b^*$ bounds the max-min value in (12) to be greater than $V_h(P_i)$ is symmetric. The conclusion follows from the max-min inequality which implies a saddle point.

Take feasible S_b. $B(S_b) \cap W(S_a^*) = \cup_{k=1}^{i} (B(S_b) \cap P_k)$. By Proposition 1 and since $B(S_b) \cap P_k$ is an upper contour subset of each P_k, for $k \leq i$ $V_h((B(S_b) \cap P_k) \leq V_h(P_k) \leq V_h(P_i)$. Thus, by Lemma 2, $V_h(\cup_{k=1}^{i} (B(S_b) \cap P_k)) \leq V_h(P_i)$. *Q.E.D.*

B.5. Proof of Proposition 3

Proof. For a given subset of strategic types $R \subset T' \times S$, let $\tilde{H}(R) \subset R \times H$, be the set of honest types that solves (8). Let the value of the problem in (7) be $\overline{V}$ with associated solutions S_a^* and S_b^*, where $R_\theta \equiv (W(S_a^*) \cap B(S_b^*)) \cap T' \times \theta$ for $\theta \in \{S, H\}$ be the associated set of strategic and honest types. First, observe that without loss in the optimization $(t, H) \in R_H \implies (t, S) \in R_S$. To see this, first suppose $v(t, H) \leq \overline{V}$, then setting $\overline{S}_b \equiv R_S \cup (R_H \setminus (t, H))$, gives $V_h(W(S_a^*) \cap B(\overline{S}_b) \geq \overline{V}$ by Lemma 2. Analogously, if $v(t, H) \geq \overline{V}$, then setting $\underline{S}_a \equiv R_S \cup (R_H \setminus \{(t, H)\})$ gives $V_h(W(\underline{S}_a) \cap B(S_b)) \leq \overline{V}$ for any feasible S_b.

Now note that setting $\overline{S}_b \equiv R_S \cup \tilde{H}(R_S)$ in (7) gives $V_h(W(S_a) \cap B(\overline{S}_b)) = \tilde{V}(R) \geq V_h(W(S_a^*) \cap B(S_b^*))$ by definition of $\tilde{H}(R)$. Let $(t, S) \in P_i$ in the ROE partition. $(t, H) \notin P_i$, then since (t, H) is isolated in $T \setminus (\cup_{j=i}^{n} P_i)$ according to $\succeq_d$, Proposition 2 implies that (t,H) is in a singleton pooling set and $\pi_h((t,H)|U^R) = v((t,H))$. *Q.E.D.*

B.6. Proof of Proposition 4

Proof. " $\Longleftarrow$ " Let $r : S \to \mathbb{R}$ be defined as $r(s) \equiv U_a^R(V_f(S), s)$ and define $E_h^r(S') \equiv \mathbb{E}[r(s')|s' \in S', s' \sim h]$ for $h \in \Delta S$. Notice that (i) $E_f^r(S) = 0$, and (ii) because U^R is strictly concave, $V_h(\tilde{S}) \geq V_f(S) \iff E_h^r(\tilde{S}) \geq 0,\ \forall \tilde{S} \subset S,\ \forall h \in \Delta S$. Because V_f is downward biased on $(S, \succeq_d)$, so is E_f^r. Because $f \geq_{ME} g$, we can order the elements of S as $(s_1, ..., s_m)$ such that $i \geq j$ implies $f(s_i)g(s_j) \geq f(s_j)g(s_i)$ and $s_j \not\succeq_d s_i$. Note that $\forall k \geq 1,\ \{s_1, ..., s_k\}$ is a lower contour subset of S and so because E_f^r is downward biased on S, $E_f^r(\{s_1, ..., s_k\}) \geq 0$. Now I will input $(s_1, ..., s_m)$ into the algorithm from Subsection 5.1 using the receiver's set valued best response as E^r instead of V.

Because the algorithm acts on a finite set and repeatedly returns strictly coarser partitions it must terminate at some stage $\overline{T}$. At this point $Q^{\overline{T}} = Q^{\overline{T}+1}$ which means that $E_f^r(Q_1^{\overline{T}}) < ... < E_f^r(Q_m^{\overline{T}})$ which implies that $E_f^r(Q_1^{\overline{T}}) = E_f^r(\{s_1, ..., s_k\}) < E_f^r(S) = 0$ if $Q^{\overline{T}}$ is non-trivial. Thus $Q^{\overline{T}} = (S)$ is the trivial partition.

Consider the partition $Q^i = (Q_1^i, ..., Q_m^i)$ generated at stage $i > 1$. Each part Q_j^i is the union of a consecutive sequence of parts from the previous partition Q^{i-1} of decreasing value. That is for each j there exists $\underline{k}(j) \leq \overline{k}(j)$ such that $Q_j^i = \cup_{l=\underline{k}(j)}^{\overline{k}(j)} Q_l^{i-1}$ and $E_f^r(Q_k^{i-1})$ is decreasing for $\underline{k}(j) \leq k \leq \overline{k}(j)$. Moreover, the likelihood ratio order is preserved at each stage, i.e. $F(Q_j^i)G(Q_{j+1}^i) \leq F(Q_{j+1}^i)G(Q_j^i)$. This means that one can use Fact 1 on the set Q_j^i, to obtain that $\forall i, j$,

$$\frac{1}{G(Q_j^i)} \sum_{l=\underline{k}(j)}^{\overline{k}(j)} E_f^r(Q_l^{i-1})G(Q_l^{i-1}) \geq \frac{1}{F(Q_j^i)} \sum_{l=\underline{k}(j)}^{\overline{k}(j)} E_f^r(Q_l^{i-1})F(Q_l^{i-1}) = E_f^r(Q_l^i).$$

Using a string of these inequalities on each part of the partition at each stage of the algorithm we get,

$$E_g^r(S) = \sum_{Q_k^1 \subset S} E_f^r(Q_k^1)G(Q_k^1) \geq \sum_{Q_k^T \subset S} E_f^r(Q_k^T)G(Q_k^T) = E_f^r(S).$$

The first equality follows from the fact that Q^1 is the complete partition on S and the second equality follows from the fact that $Q^T = (S)$ is the trivial partition on S.

" $\implies$ " Suppose there exists a lower contour subset $L = W(L) \subset S$, such that $V_f(L) < V_f(S) \implies V_f(L) < V_f(S \setminus L)$. Define $g(s) = \frac{f(s)}{F(L)}$ if $s \in L$ and $g(s) = 0$ otherwise. $f \geq_{ME} g$ but $V_f(S) > V_g(S)$. *Q.E.D.*

B.7. Proof of Theorem 1

Proof. " $\implies$ " Note that if $f \geq_{ME} g$ then $Z^g \equiv \{t \in T : g(t) = 0\}$ is an upper contour set of $(T, \succeq_d)$ and $Z^f \equiv \{t \in T : f(t) = 0\}$ is a lower contour set of $(T, \succeq_d)$. Let $P^g = (P_1^g, ..., P_l^g, Z^g)$ be the ROE partition under g and take arbitrary j and $t \in P_j^g$. Define $D^g \equiv \cup_{k=1}^{j} P_k^g$ and consider the problem,

$$\max_{\tilde{S} \subset T:\ B(\tilde{S}) \cap (D^g \setminus Z^f) \neq \emptyset} V_f(B(\tilde{S}) \cap (D^g \setminus Z^f)), \tag{13}$$

with corresponding solution $\overline{S}$ with $\overline{R} \equiv B(\overline{S}) \cap (D^g \setminus Z^f)$. Note that $\overline{R} \subset \text{Supp}(f) \cap \text{Supp}(g)$. Because $D_g \setminus Z^f$ is a feasible S_a in Theorem 2 under f, $V_f(\overline{R}) \geq \pi_f(t|U^R)$. By Lemma 1 V_f is downward biased on $\overline{R}$. Thus by Proposition 4, this means that $V_g(\overline{R}) \geq V_f(\overline{R})$. Now notice that by Proposition 2,

$$\pi_g(t|U^R) = \max_{\tilde{S} \subset T:\ B(\tilde{S}) \cap D^g \neq \emptyset} V_g(B(\tilde{S}) \cap D^g).$$

Moreover, because Z^f is a lower contour subset of T,

$$\max_{\tilde{S} \subset T:\ B(\tilde{S}) \cap D^g \neq \emptyset} V_g(B(\tilde{S}) \cap D^g) \geq \max_{\tilde{S} \subset T:\ B(\tilde{S}) \cap (D^g \setminus Z^f) \neq \emptyset} V_g(B(\tilde{S}) \cap (D^g \setminus Z^f)).$$

Thus $\pi_g(t|U^R) \geq V_g(\overline{R})$. Putting this string of inequalities together gives the desired conclusion that $\pi_f(t|U^R) \leq \pi_g(t|U^R)$.

" $\impliedby$ " Let $t \succeq_d t'$ and $f, g \in \Delta T$ such that $f(t)g(t') < f(t')g(t)$ with $t' \in I$. Define $S \equiv W(\{t\}) \cap B(\{t'\})$, and $\tilde{S} \equiv S \setminus \{t, t'\}$. Notice that either (a) $\frac{F(\tilde{S} \cup \{t\})}{f(t')} < \frac{G(\tilde{S} \cup \{t\})}{g(t')}$, or (b) $\frac{f(t)}{F(\tilde{S} \cup \{t'\})} < \frac{g(t)}{G(\tilde{S} \cup \{t'\})}$. Consider two actions $\overline{v} > \underline{v}$ and let U^R be quadratic loss, with (i) $v(s) = \overline{v}\ \forall s \notin W(\{t\})$, (ii) $v(s) = \underline{v}\ \forall s \in W(t) \setminus B(t')$, (iii) in case (a) $v(s) = \overline{v}\ \forall s \in \tilde{S}$ in case (b) $v(s) = \underline{v}\ \forall s \in \tilde{S}$, (iv) $v(t) = \underline{v}$, and (v) $v(t') = \overline{v}$.

For any distribution $h \in \Delta T$, the ROE partition is made up of 3 parts given by $P_1 = W(\{t\}) \setminus S$, $P_2 = S$ and $P_3 = T \setminus W(\{t\}))$. The associated actions (as long as each part has positive support are $V_h(P_1) = \underline{a}$, $V_h(P_3) = \overline{a}$, and $V_h(P_2) = V_h(S)$. $V_f(S) > V_g(S)$ in either case. Thus $\pi_f(t'|U^R) > \pi_g(t'|U^R)$. *Q.E.D.*

B.8. Proof of Proposition 5

Proof. Recall that given a period 1 sender strategy $\sigma : T \to \Delta T$, $h_s \in \Delta T$ is the receiver's interim belief following period 1 disclosure s. Note that

$$h_s(t_2) = \frac{\sum_{t_1 \in W(\{t_2\})} \frac{\tilde{h}(t_2)}{\tilde{H}(B(t_1))} \sigma_{t_1}(s) h_1(t_1)}{\sum_{t_1 \in T} \sigma_{t_1}(s) h_1(t_1)}. \tag{14}$$

Claim 2. *Suppose $s', s'' \in \cup_{t_1 \in T} \operatorname{Supp}(\sigma_{t_1})$, $S \subset T$ is an interval, $S \cap \operatorname{Supp}(h_{s'}) \neq \emptyset$, and $S \cap \operatorname{Supp}(h_{s''}) \neq \emptyset$. If $\sigma_{t_1}(s'') = 0$ $\forall t_1 \in S$ then $h_{s'} \geq_{ME} h_{s''}$ with on $(S, \succeq_d)$.*

Proof of Claim: Take $\bar{t}_2, \underline{t}_2 \in \operatorname{Supp}(h_s)$ for some period 1 declaration s such that $\bar{t}_2 \succeq_d \underline{t}_2$. It holds that $h_s(\bar{t}_2)/\tilde{h}(\bar{t}_2) \geq h_s(\underline{t}_2)/\tilde{h}(\underline{t}_2)$. To see this expand the expressions on either side of the inequality using (14) to get

$$\sum_{t_1 \in W(\bar{t}_2)} \frac{\sigma_{t_1}(s_1'') h_1(t_1)}{\tilde{H}(B(t_1))} \geq \sum_{t_1 \in W(\underline{t}_2)} \frac{\sigma_{t_1}(s_1'') h_1(t_1)}{\tilde{H}(B(t_1))}. \tag{15}$$

Since $\bar{t}_2 \succeq_d \underline{t}_2$, the LHS sums more terms (in a set containment sense) and is therefore greater than the RHS. By showing that the above is an equality for $s = s''$, we will have shown that $h_{s'}(\bar{t}_2) h_{s''}(\underline{t}_2) \geq h_{s'}(\underline{t}_2) h_{s''}(\bar{t}_2)$, i.e. $h_{s'} \geq_{ME} h_{s''}$ on S. Suppose to the contrary that the LHS $>$ RHS of (15) for $s = s''$. This means there exists $t_1 \in W(\{\bar{t}_2\}) \setminus W(\{\underline{t}_2\})$ such that $\sigma_{t_1}(s'') > 0$. But since $t_1, \underline{t}_2 \in W(\bar{t}_2)$ and since $\underline{t}_2 \not\succeq_d t_1$, by the UEPP $t_1 \succeq_d \underline{t}_2$. But because S is an interval, $t_1 \in S$. This means that $\sigma_{t_1}(s'') = 0$ by assumption, which is a contradiction.

I first show that under the UEPP the only equilibrium allocation is static communication. Consider a candidate non-degenerate equilibrium allocation $\tilde{\pi} : T \to \Delta A$ with associated sender period 1 strategy $\sigma : T \to \Delta T$ and with corresponding period 2 ROE partitions given period 1 disclosure s of $B(\{t_1 : \sigma_{t_1}(s) > 0\})$ denoted as $(P_1^s, ..., P_{m(s)}^s)$.[34] I will show that $\forall t \in \operatorname{Supp}(h_{s'}) \cap \operatorname{Supp}(h_{s''})$, we have that (i) $t \in P_k^{s'} \cap P_j^{s''} \implies P_k^{s'} \cap \operatorname{Supp}(h_{s''}) = P_j^{s''} \cap \operatorname{Supp}(h_{s'})$ and (ii) $\pi_{h_{s'}}(t|U^R) = \pi_{h_{s''}}(t|U^R)$.

[34] if $\tilde{\pi}$ is degenerate then the result follows from the fact the degenerate action must be the best response to each pooled set under the unconditional period 2 distribution by Lemma 2.

Towards a contradiction, let $P_k^{s'}$ be the highest index part of $P^{s'}$ to violate condition (i) or (ii) or both. Take $P_j^{s''}$ with the highest index j such that $P_j^{s''} \cap P_k^{s'} \neq \emptyset$. This means that $P_{j'}^{s''} \cap P_k^{s'} = \emptyset \ \forall j' > j$ and since higher parts of $P^{s'}$ cannot violate (i), $P_j^{s''} \cap P_{k'}^{s'} = \emptyset \ \forall k' > k$. This means that $P_k^{s'}$ is feasible in the maximization that selects $P_j^{s''}$ in the construction of each partition of the maximization version of Algorithm 1. At the stage where $P_k^{s'}$ is selected, the fact that $P_j^{s''} \cap P_{k'}^{s'} = \emptyset \ \forall k' > k$ means that $P_j^{s''}$ is also available. This means that $V_{h_{s'}}(P_k^{s'}) \geq V_{h_{s'}}(P_j^{s''})$ and $V_{h_{s''}}(P_j^{s''}) \geq V_{h_{s''}}(P_k^{s'})$.

Now suppose that $V_{h_{s'}}(P_k^{s'}) > V_{h_{s''}}(P_j^{s''})$. Each period 1 type $t_1 \in P_k^{s'}$ expects to remain in $P_k^{s'}$ in period 2 with strictly positive probability in which case he gets a strictly higher action from s', or to end up in a higher part in which case he is indifferent between s' and s'' by assumption. This means that declaring s' strictly dominates declaring s'' for t_1 and so $\sigma_{t_1}(s'') = 0 \ \forall t_1 \in P_k^{s'}$. Thus by Claim 2 $h_{s'} \geq_{ME} h_{s''}$ on $P_k^{s'}$. Using Proposition 4, this implies that $V_{h_{s'}}(P_k^{s'}) \leq V_{h_{s''}}(P_k^{s'})$. Using the inequality above that $V_{h_{s''}}(P_j^{s''}) \geq V_{h_{s''}}(P_k^{s'})$ leads to a contradiction. The opposite case in which $V_{h_{s'}}(P_k^{s'}) < V_{h_{s''}}(P_j^{s''})$ is symmetric.

The only remaining case is that $V_{h_{s'}}(P_k^{s'}) = V_{h_{s''}}(P_j^{s''})$. In this case $P_k^{s'}$ does not violate condition (ii) so it must violate condition (i) by assumption. Without loss, let $P_k^{s'} \not\subset P_j^{s''}$, the opposite case being symmetric. Now define the non-empty interval $R \equiv \cup_{j'<j} P_{j'}^{s''} \cap P_k^{s'}$. Notice that period 1 types in R strictly prefer declaration s' to s'': in period 2 they either remain in R in which case s' is preferred or they end up in a higher part in which case they are indifferent by assumption. This means that $\sigma_{t_1}(s'') = 0 \ \forall t_1 \in R$. Thus by Claim 2 $h_{s'} \geq_{ME} h_{s''}$ on R.

Since by assumption $P_{j'}^{s''} \cap P_{k'}^{s'} = \emptyset \ \forall k' > k, \ j' < j$, the upper contour subset of $P_{j'}^{s''}$ given by $(\cup_{k' \geq k} P_{k'}^{s'}) \cap P_{j'}^{s''} = P_k^{s'} \cap P_{j'}^{s''}$. Thus by Proposition 1 $V_{h_{s''}}(P_k^{s'} \cap P_{j'}^{s''}) \leq V_{h_{s''}}(P_{j'}^{s''}) < V_{h_{s''}}(P_j^{s''}) \ \forall j' < j$. R is the disjoint union of these upper contour subsets and so by Lemma 2 $V_{h_{s''}}(R) < V_{h_{s''}}(P_j^{s''})$. By analogous logic R is a lower contour subset of $P_k^{s'}$. This means that any lower contour subset of R, denoted $\underline{R}$, is in turn a lower contour subset of $P_k^{s'}$ and thereby has $V_{h_{s'}}(\underline{R}) \geq V_{h_{s'}}(P_k^{s'})$. The following claim extending Proposition 4 establishes a contradiction using $S = R$, $h_{s'} = f$ and $h_{s''} = g$ and $\overline{v} = V_{h_{s'}}(P_k^{s'})$. This completes the proof.

Claim 3. *Suppose that $(S, \succeq_d)$ is a disclosure ordered subset. Take two distributions $f, g \in \Delta S$ such that $f \geq_{ME} g$. If $\overline{v}$ is such that, $V_f(W(S') \cap S) \geq \overline{v} \ \forall S' \subset S$, then $V_g(S) \geq \overline{v}$.*

Proof of Claim: Suppose not, i.e. $V_g(S) < \overline{v}$. There exists a lower contour subset $\underline{W} \subset S$ such that (i) $V_g(\underline{W}) < \overline{v}$, and (ii) $V_g(W(S') \cap \underline{W}) \geq \overline{v}\ \forall S' :\ \underline{W} \not\subset W(S')$. To see that such a subset exists, notice first that S satisfies (i). Furthermore if a lower contour subset W satisfies (i) but not (ii), then the violator to (ii) — W' — is a strictly smaller lower contour subset that also satisfies (i). Since S is finite this process must terminate with a lower contour subset satisfying both properties.

By construction $\underline{W}$ is a downward biased set under g. Thus by Proposition 4 $V_g(\underline{W}) \geq V_f(\underline{W})$ but by assumption $V_f(\underline{W}) \geq \overline{v}$, a contradiction. *Q.E.D.*

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