Evidence and Skepticism in Verifiable Disclosure Games*

Daniel Rappoport†

September 2, 2022

Abstract

A key feature of communication with evidence is skepticism: a receiver will attribute any incomplete disclosure to the sender concealing unfavorable evidence. I study when a change in the receiver's prior belief about the sender's evidence induces more skepticism, i.e. induces the receiver, regardless of his preferences, to take an equilibrium action that is less favorable for the sender following every message. I provide a definition of when one receiver prior belief expects more evidence than another and show that this characterizes more skepticism. As an input, I fully characterize receiver optimal equilibrium outcomes in general verifiable disclosure games.

*I am deeply indebted to Navin Kartik and Andrea Prat for their guidance on my dissertation. I thank Charles Angelucci, Ashna Arora, Yeon-Koo Che, Alex Frankel, Jonathan Glover, Ilan Guttman, Marina Halac, Johannes Horner, Jacob Leshno, Elliot Lipnowski, Emir Kamenica, Nate Neligh, Pietro Ortoleva, Doron Ravid, Valentin Somma, Amir Ziv, Weijie Zhong, and the audience at the Columbia microeconomic theory colloquium for their comments. All errors are my own.

†Booth School of Business, University of Chicago. 5807 S Woodlawn Ave office 430, Chicago, IL 60637. Email: daniel.rappoport@chicagobooth.edu.

1. Introduction

Communication during criminal trials, financial audits, and investment pitches is often verifiable. In these settings, communication is less about the risk of misrepresentation (cheap talk) and more about which evidence is presented or omitted (disclosure). Any rational observer (receiver) is naturally *skeptical* of the evidence presented by an interested party (sender): the receiver will partially attribute incomplete disclosures to the sender concealing unfavorable evidence.

This skepticism is harmful to the sender: a prosecutor would always prefer to be faced with a less skeptical juror, and an entrepreneur a less skeptical investor. It is also natural that the receiver's beliefs about the sender's evidence modulate his degree of skepticism. Indeed, the criminal justice literature identifies a "CSI effect": Shelton et al. (2009) find that jurors who are more informed about forensics are less likely to convict given the same evidence profile. However, it is not clear which beliefs induce more skepticism than others. The main goal of this paper is to characterize this comparison, i.e. to identify the sender's preferences over the receiver's prior beliefs. Understanding these preferences is important because the sender can often influence the receiver beliefs he faces. For example:

1. During jury selection, a prosecutor questions potential jurors in order to identify those that will return the highest probability of conviction. The prosecutor selects jurors on many criteria including their beliefs about the evidence available. The prosecutor wants to choose jurors who hold the least skeptical beliefs concerning his evidence. Which juror beliefs will achieve this goal?

2. During each investment round, an entrepreneur discloses customer reviews, prototypes, and sales numbers to an investor in order to obtain funding. There are generally multiple rounds of investment. Thus, the entrepreneur will be concerned about how an early disclosure affects the skepticism of future investors. Which beliefs will the entrepreneur want to induce in these future investors, and which early disclosures will achieve this goal?

The common features of these examples are that a sender (e.g. prosecutor) interacts with a receiver (e.g. jury) through both verifiable disclosure (e.g. the criminal trial), and some preliminary action (e.g. jury selection). Each preliminary action will induce different receiver beliefs going into the verifiable disclosure game. Thus, a key issue is understanding the sender's preferences over these beliefs within the static verifiable disclosure framework.

One intuition is that the receiver's degree of skepticism increases when he expects more evidence to be available. This accords with the CSI effect: prosecutors will try to avoid jurors with bullish views about the amount of evidence that can be presented. The issue is that there are potentially multiple dimensions over which the sender can be informed, for example a prosecutor can have access to DNA evidence, witness testimony, other forensic evidence, or any subset of these. This makes it difficult to even define what it means to believe the sender has more evidence. Indeed, there do not exist general comparative statics or characterization results in multidimensional verifiable disclosure models.

I study a general verifiable disclosure model in which a sender communicates with a receiver in order to persuade him to take a high action.[1] The sender's set of potential evidence realizations, or "type space", doubles as the set of feasible messages. The messages available to each type are governed by a partial *disclosure order*: if one sender type dominates another according to the disclosure order, the former can make a declaration to the receiver that he is the latter type. For example, a prosecutor with DNA evidence dominates a prosecutor with no evidence according to the disclosure order as the former can masquerade as the latter through omission. In the context of arbitrarily complex evidence structures, I provide a definition of when one evidence distribution has "more evidence" than another: if one sender type dominates another according to the disclosure order, the former is relatively more likely than the latter under a distribution with more evidence.

Focusing on the receiver optimal equilibrium, Theorem 1 establishes that, regardless of the true distribution of evidence and the preferences of the receiver he faces, the sender *always* wants to be thought of as having less evidence. Moreover, the converse is also true. That is, if the sender prefers to induce one receiver belief over another in the broad sense above, then the latter belief must have more evidence than the former.

My model makes no assumptions on the relationship between the amount of evidence the sender has and its value, i.e. how high an action it would induce from the receiver if truthfully revealed. Instead the relationship between more evidence and more skepticism emerges from the receiver optimal equilibrium structure. My second main contribution is to fully characterize the receiver optimal equilibrium.

[1] The receiver chooses an action in $\mathbb{R}$.

Proposition 1 provides necessary and sufficient conditions for a set of sender types to "pool together", i.e. obtain the same equilibrium outcome. This leads to an explicit expression for the receiver optimal equilibrium allocation in Theorem 2. At a high level, the pooled set for a given sender type forms through simultaneously minimizing the receiver's value over types that choose to mimic him and maximizing the receiver's value over the types that he chooses to mimic.

Signaling to affect receiver beliefs and decrease skepticism is at the heart of many disclosure papers. As an illustration, Subsection 6.1 introduces a simple two period verifiable disclosure model. Corollary 1, an implication of the main result, shows that if the sender induces different beliefs in period 2 that are ranked by the more evidence relation, then the associated period 1 action must compensate the sender for inducing the more evidence belief. I then show how models in dynamic disclosure papers such as Guttman et al. (2014), and Grubb (2011) are examples of the model in Subsection 6.1, and how their results make use of specific versions of Corollary 1. These papers establish and/or use seemingly different results within the static disclosure framework that rank the skepticism of different receiver beliefs. My results unify these tools as examples of the more evidence – more skepticism equivalence, as well as facilitate analysis of new signaling questions which involve multidimensional evidence structures.

As proof of concept, I study two dynamic disclosure applications. The first application is an extension of the repeated disclosure model in Grubb (2011) to multidimensional evidence structures. I show that the action for non-disclosure in period 1 decreases with how much the sender discounts period 2. The reason is that the sender values a reputation for having less evidence which can be achieved through non-disclosure. I also explain why certain other results in repeated disclosure do not extend to multidimensional evidence structures.

The second application considers an entrepreneur who obtains and discloses evidence gradually in order to persuade an investor. I examine whether the investor can benefit from additional early communication relative to the game in which communication only occurs right before the investment decision. The potential for informative early communication relies on the multidimensional evidence structure: in the Dye model, there is no residual uncertainty after a single disclosure. Proposition 5 shows that the receiver does not benefit from early communication regardless of his preferences or prior beliefs if and only if the evidence structure satisfies what I term the "Unique Evidence Path Property" (UEPP). The interpretation of the UEPP is that the current evidence reveals the sequence of previous investigations undertaken, e.g. the performance test results for a prototype can only be revealed if the prototype is first developed. Roughly, under the UEPP, any informative early disclosure signals that the sender has more evidence which, in light of Theorem 1, incentivizes the sender to avoid such a disclosure.

Layout The paper proceeds as follows. Subsection 1.1 discusses the related literature. Section 2 presents the model and lists examples that fit my framework. Section 3 defines the more skepticism and more evidence orders and states the main result that they are equivalent. Section 4 and Section 5 provide the analysis necessary to establish the main result. First I characterize the receiver optimal equilibrium in Section 4, and then I introduce the comparative statics techniques in Section 5. Section 6 illustrates how the main result is useful in dynamic disclosure: Subsection 6.1 introduces a two period workhorse model, and details how many dynamic disclosure examples from the literature fit this framework. The next two sections introduce the dynamic disclosure applications. Section 7 extends the main results to consider senders who have some (potentially evidence dependent) probability of preferring lower actions, or being unbiased. Omitted proofs are in the appendix.

1.1. Related Literature

The first verifiable disclosure models were introduced by Milgrom (1981), Grossman (1981), and Grossman and Hart (1980). In these models, the sender could be vague about his private information but not lie, i.e. he could declare any subset of states that contains the true state. The main finding is the "unraveling" result that in any equilibrium the sender fully reveals his information. There are many ways unraveling can fail: if the sender's direction of bias depends on his type (e.g. Seidmann and Winter (1997)), if the sender pays a cost to disclose information (e.g. Verrecchia (1983)), or if the receiver is uncertain about the sender's information endowment which is the focus of this paper (e.g. Dye (1985) and Jung and Kwon (1988)).[2,3]

Shin (2003) and Dziuda (2011) consider multidimensional versions of the Dye evidence framework in which the sender obtains potentially multiple pieces of ei-

[2] Hagenbach et al. (2014) and Mathis (2008) provide necessary and sufficient conditions for unraveling in a general framework.

[3] For surveys of this literature see Milgrom (2008) and Dranove and Jin (2010).

ther good or bad evidence and can disclose any subset.[4] The analysis in these models is simplified by the observation that (under mild distributional assumptions) the sender always plays a "sanitization" strategy of concealing bad pieces and fully revealing good pieces. I study more complex evidence structures in which there is a more fragile dependence of the equilibrium pooled sets on the parameters of the model.

As mentioned above, there have also been a number of dynamic extensions. Guttman et al. (2014) and Acharya et al. (2011) consider dynamic frameworks in which the receiver is also uncertain about *when* the sender has obtained evidence. Grubb (2011) considers a repeated disclosure game in which there is a persistent and privately known evidence distribution. In subsubsection 6.1.1 I discuss the relationship between the comparative statics results used in these papers and my main result.

Another strand of literature shows that the receiver's utility in some equilibrium of the verifiable disclosure game is the same as that in which the receiver can commit to a best response before learning the sender's message. This equivalence was first introduced in Glazer and Rubinstein (2004) and further explored by Sher (2011), and Ben-Porath et al. (2017) in the context of multiple senders. Hart et al. (2017) identifies the equilibrium that achieves this equivalence through the "truth leaning refinement".[5] I focus on this receiver optimal equilibrium and my model is the same as that in Hart et al. (2017).

In addition to the above equivalence results, Glazer and Rubinstein (2004) and Sher (2014) derive methods to find the receiver optimal equilibrium. However, their models involve a binary action choice and only two types of senders - acceptable and unacceptable. Bertomeu and Cianciaruso (2016) also propose an algorithm for solving verifiable disclosure games when pure strategy equilibria exist. My approach focuses on equilibrium outcomes instead of sender strategies. This permits tractable analysis despite the fact that sometimes (generically) verifiable disclosure games only admit mixed strategy equilibria.

[4] Dziuda (2011) also consider uncertainty over the preferences of the sender and over whether he is honest or strategic. Einhorn (2007) also considers this kind of uncertainty.

[5] Hart et al. (2017) also extends the commitment equivalence result.

2. Model

The setting involves a single sender and a single receiver. The sender observes his type $t \in T$ which constitutes his private information, where $|T| = n$. The sender then sends a message from his feasible set (described below). The receiver observes this message and then chooses an action $a \in A \equiv \mathbb{R}$. The receiver has a prior belief $h \in \Delta T$ over the sender's type with associated measure $H : 2^T \to [0, 1]$.[6] The sender may have some alternative prior belief over his type but it is not relevant to the results. This section lays out and discusses the specific assumptions on preferences, messaging, and equilibrium focus, and shows how typical verifiable disclosure examples fit into this framework.

Preferences I assume that the sender's utility $U^S(a, t)$ is strictly increasing in the action a for every type t.[7] The receiver's utility, $U^R : A \times T \to \mathbb{R}$, depends on both the action and the sender's private information. I assume that U^R is strictly concave and differentiable in a, and that $U^R(a, t)$ admits a maximum in a for each $t \in T$. I denote the set of all such receiver utilities as Υ. These assumptions imply that neither party will randomize over induced actions in equilibrium, and so to ease notation I identify the sender's utility with the action taken, i.e. $U^S(a, t) \equiv a$.[8]

Denote the receiver's unique best response to type t by $\boldsymbol{v(t)} \equiv \arg\max_a U^R(a, t)$. Similarly, define $\boldsymbol{V_h(S)} \equiv \arg\max_a \mathbb{E}[U^R(a, t)|t \in S, t \sim h]$ to be the receiver's best response conditional on the sender's type being in S and distributed according to h. I refer to sets of types with relatively high (low) optimal actions, as "high (low) value". A leading example is when the receiver's utility is quadratic loss, i.e. $U^R(a, t) = -(a - v(t))^2$ for some function $v : T \to \mathbb{R}$. In this case $V_h(S) = \mathbb{E}[v(t)|t \in S, t \sim h]$ is the conditional expectation of the receiver's type-dependent value.

Messaging Technology I follow Hart et al. (2017) and assume that the message space is the type space with the interpretation that type t sending message t' is

[6] In general, I refer to distributions with lower case and their associated measures with upper case.

[7] In Section 7 I show that the analysis can be adapted to when the sender has a (potentially type dependent) probability of preferring lower actions, or having the same preferences of the receiver.

[8] The strict concavity ensures that the receiver has a unique optimal action for all distributions over sender types. In combination with the assumption that the sender's utility is strictly increasing in a for all t, this implies that the sender will never randomize over messages which induce different actions. An implication is that all sender utilities which have the same ordinal preferences over deterministic actions as $U^S(a, t) = a$ will admit the same equilibria.

type t "mimicking" t', i.e. making the declaration "I am type t'" to the receiver. I assume that there is a partial order $\boldsymbol{\succeq_d}$ over T, referred to as the *disclosure order*. $t \succeq_d t'$ means that t can send message t', i.e. the set of available messages to each type t is given by $\{t' : t \succeq_d t'\}$.[9] The partial order assumption imposes reflexivity, transitivity, and antisymmetry. That is, (i) every type can truthfully reveal (send their corresponding message), (ii) if t can mimic t' and t' can mimic t'', then t can mimic t'', and (iii) if t can mimic t', t' cannot mimic t.[10]

Strategies and Equilibrium A strategy for the sender is $\sigma : T \rightarrow \Delta T$ where $\text{Supp}(\sigma_t) \subset \{t' : t \succeq_d t'\}\ \forall t$. I denote $\sigma_t(s)$ as the probability that the sender of type $t \in T$ sends message $s \in T$. Because the receiver's utility is strictly concave, it is without loss to restrict the receiver to use a pure strategy, $a : T \rightarrow A$, which specifies an action choice in response to each message.

A perfect Bayes equilibrium (PBE) is a pair of strategies for the sender and receiver such that (i) $\sigma_t(s) > 0 \implies a(s) = \max\{a(s') : t \succeq_d s'\}$, (ii) $a(s) = \arg\max_a \mathbb{E}[U^R(a,t)|\sigma, s]\ \forall s \in \cup_{t\in T} \text{Supp}(\sigma_t)$, and (iii) $a(s) = V_q(\{t' : t' \succeq_d s\})$ for some $q \in \Delta\{t : t \succeq_d s\}\ \forall s \in T$. In words: (i) the sender only sends messages that give him the highest action within his feasible set, (ii) the receiver best responds to each on path message where he updates according to Bayes rule given the sender's strategy, and (iii) for off path messages the receiver best responds to some belief over sender types that could feasibly send this message.

Because the sender strictly prefers higher actions, and the receiver's utility is strictly concave over actions, each sender type must be allocated a deterministic action in any PBE. I focus on the receiver optimal PBE which I henceforth refer to as the **ROE**. Denote $\boldsymbol{\pi_h(t|U^R)} \in A$ as the ROE allocation when the receiver has prior

[9] The verifiable disclosure literature sometimes uses a more primitive but equivalent set of messaging assumptions. There is an arbitrary message space M and each type has access to some subset $E(t) \subset M$, where the message correspondence E satisfies a "normality" (Bull and Watson (2004)) or "nested range condition" (Green and Laffont (1986)). A message structure is normal if $\forall t \in T$, there exists $e_t \subset E(t)$, such that $\forall t, t' \in T$, $e_t \in E(t') \implies E(t) \subset E(t')$. Given a normal message structure, the following disclosure ordered type space $(T, \succeq_d)$ has the same set of equilibrium outcomes, i.e. mappings from types to actions. $T \equiv M$ with $Pr(e_t) \equiv Pr(t)$ and $Pr(m) = 0$ otherwise, and $e_t \succeq_d m \iff m \in E(t)$ with no other relations.

[10] The reflexivity and transitivity are substantive assumptions, while the antisymmetry is without loss in the following sense. Consider that in addition to disclosable evidence from $(T, \succeq_d)$, the sender also has non-verifiable private information given by $\theta \in \Theta$ so that the type space is $T \times \Theta$. Messaging could be modeled by the preorder $\succeq_d'$ defined by $(t,\theta) \succeq_d' (t',\theta') \iff t \succeq_d t'$. However, because the sender always prefers higher actions, types within an equivalence class of such a pre-order cannot induce different equilibrium actions. Thus we can think of each type as the representative of its equivalence class.

belief h and utility function U^R, and the sender is type $t \in \text{Supp}(h)$. ROE strategies are not unique and are kept in the background of the analysis.

While there is a unique PBE in the Dye model, multidimensional evidence models will generally have multiple PBE. As discussed in Section 4, these unrefined PBE put very little structure on the equilibrium pooled sets. A number of studies have provided justifications for focusing on the ROE. Hart et al. (2017) shows that the truth leaning refinement, in which (i) the receiver interprets each off path message credulously, and (ii) the sender is truthful when doing so maximizes his obtained action, selects the ROE.[11] Relatedly, Bertomeu and Cianciaruso (2016) shows that the ROE is also the unique equilibrium without "self signaling sets".[12] Finally, Sher (2011) and Hart et al. (2017) establish that the ROE is equivalent to the mechanism design outcome, where the principal can commit to an allocation as a function of the sender's disclosed evidence. Thus, the proceeding results are also applicable to the commitment problem.

2.1. Examples

Common examples that fit the framework are described below. I represent $(T, \succeq_d)$ as a directed graph with vertices representing types and directed paths representing dominance in the disclosure order.

Dye Evidence and Multi-dimensional Dye Evidence Assume there are k potential kinds of evidence, e.g. a prosecutor can acquire DNA evidence, witness testimony, and blood test results. Each kind of evidence has realizations in some finite set $E^i = \{e^i_\emptyset, e^i_1, ..., e^i_{m^i}\}$ where $e^i_\emptyset$ is interpreted as not acquiring the i'th kind of evidence. For example, the prosecutor's DNA test could fail to yield a result along with providing a confirmation or exoneration. The sender's type $t \in E^1 \times ... \times E^k$ refers to his set of evidence. The disclosure order is given by $(y^1, ..., y^k) \succeq_d (z^1, ..., z^k) \iff z^i \in \{y^i, e^i_\emptyset\} \, \forall i$. The interpretation is that each type of sender can report or withhold any subset of pieces of evidence in his possession. The right panel of Figure 1 illustrates a multidimensional evidence model where $k = 2$ and $E^1 = E^2 = \{e^i_\emptyset, 0, 1\}$ where each type is referred to as its set of

[11] Hart et al. (2017) also show that perturbations where the sender has some small probability of being honest have a unique equilibrium outcome that is arbitrarily close to the ROE.

[12] Given an equilibrium, a self signaling set is defined as a set of sender types for which the receiver's best response would incentivize exactly that set of sender types to deviate from the equilibrium.

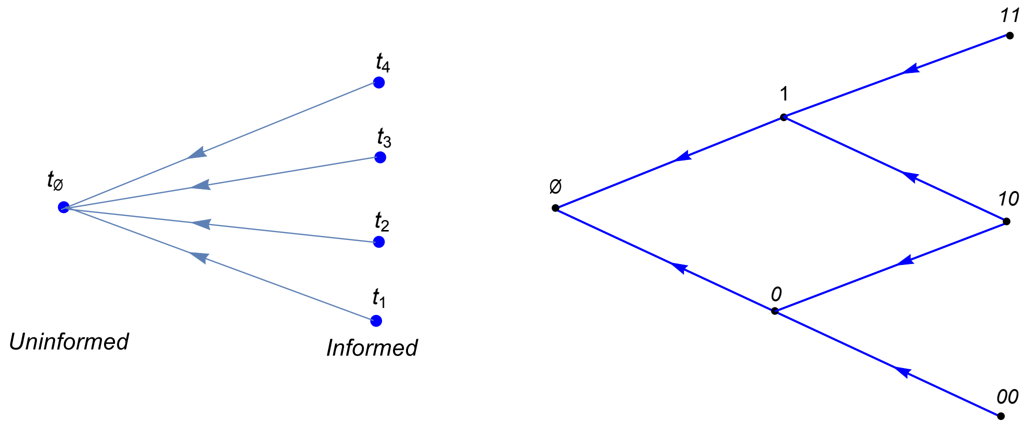
non-empty evidence acquisitions.

The special case in which $k = 1$ is called the "Dye evidence model" and has been widely used in the verifiable disclosure literature.[13] The Dye evidence model involves a no evidence type, $t_\emptyset$ who cannot prove he is uninformed, and some evidence types $\{t_1, ..., t_{n-1}\}$ who can either completely reveal or pretend to be uninformed. The left panel of Figure 1 illustrates the disclosure order in the Dye model.

Vagueness There is some state of the world $x \in X$ drawn from $h \in \Delta X$. The type space is all non-empty subsets of X, i.e. $T = 2^X \setminus \{\emptyset\}$. The disclosure order is given by $t \succeq_d t' \iff t \subset t'$. The interpretation is that each type t can credibly reveal himself or be "vague". For example, it is easy to hold a manager accountable for misreporting earnings, but more difficult when he is imprecise about these earnings. I illustrate the case in which $X = \{0, 1\}$ in the left panel of Figure 2. Type $\{0\}$ can either truthfully report $\{0\}$, or be vague and report $\{0, 1\}$, however she cannot "lie" and report $\{1\}$. In this example, and in the classic examples from Grossman (1981) and Milgrom (1981), the positive probability types are the singletons of X, i.e. the sender completely learns the state. This description uses zero probability types to model additional messaging options for positive probability types. Any non-singleton type t is zero probability, but represents a feasible message for types $t' \subset t$. However the general vagueness model above also captures situations in which the sender only learns partial information about the state, i.e. some non-singleton type t is positive probability.

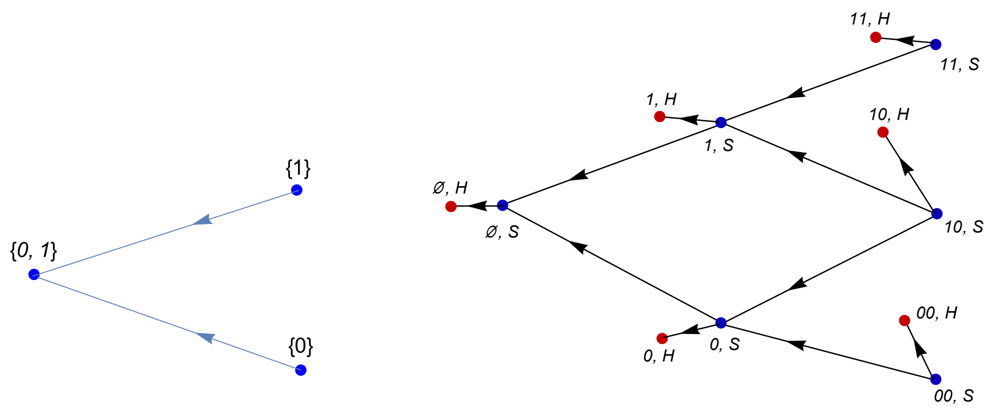
Honest Types In addition to obtaining evidence from some T', the sender can either be strategic, S, or honest, H. Strategic types can disclose evidence according to some arbitrary disclosure order $\succeq_d'$, while honest types must truthfully reveal. The total type space and disclosure order are denoted by $(T, \succeq_d)$ and defined as follows: $T \equiv T' \times \{S, H\}$ and $t \succeq_d' t' \implies (t, S) \succeq_d (t', \theta')\ \forall \theta' \in \{S, H\}$ and $\succeq_d$ admitting no other non-reflexive comparisons. Figure 2 displays the multidimensional example from the right panel of Figure 1 with the addition of honest types. I use the methods established in Section 4 to further solve the honest types model in Subsection 7.2.

[13] Examples include Grubb (2011), Acharya et al. (2011), and Bhattacharya and Mukherjee (2013).



(a) Dye Model (b) Multidimensional Evidence

Figure 1: Common Examples



(a) Vagueness (b) Honest Types

Figure 2: Other Examples

Complete Order and Empty Order The cases in which the disclosure order is complete or empty serve as illustrative examples. A completely disclosure-ordered type space is given by $(T, \succeq_d)$, with $T = \{t_1, ..., t_n\}$, and $i \geq j \iff t_i \succeq_d t_j$. That is, types with higher indices can report all types with lower indices. An empty disclosure ordered type space $(T, \succeq_d)$ is given by $t \succeq_d t' \iff t = t'$.

3. Characterizing Increased Skepticism

As stated in the introduction, the goal is to understand the sender's preferences over receiver prior beliefs in the general verifiable disclosure model. My main result, Theorem 1, characterizes when this preference holds regardless of his realized type and the receiver's utility function.

3.1. More Skepticism and More Evidence

Definition 1. Let $f, g \in \Delta T$ with $I \equiv \text{Supp}(f) \cap \text{Supp}(g)$. f induces **more skepticism** than g, also expressed as $f \geq_{MS} g$, if

$$\pi_f(t|U^R) \leq \pi_g(t|U^R) \quad \forall t \in I, \ \forall U^R \in \Upsilon. \tag{1}$$

One prior belief induces more skepticism than another if it leads *any* receiver to take a lower ROE action for every type.[14] Note that because the sender prefers higher actions, the sender is worse off, no matter his ex-ante distribution, when facing a more skeptical receiver.

The more skepticism order is difficult to compute. In attempting to compare two distributions, one would have to compute the corresponding ROE actions for every type and every receiver utility. This motivates defining the following primitive order.

Definition 2. Let $f, g \in \Delta T$. f has **more evidence** than g with respect to $(T, \succeq_d)$, also expressed as $f \geq_{ME} g$, if

$$\forall t, t' \in T, \quad t \succeq_d t' \implies f(t)g(t') \geq f(t')g(t). \tag{2}$$

For any type t that can mimic t', t is relatively more likely than t' under a prior distribution with more evidence. With the view that each type is a set of evidence which the sender can present (as in multidimensional Dye), the more evidence relation shifts probability to types that literally have more pieces of evidence. If $\succeq_d$ were a complete order, then $f \geq_{ME} g$ would be equivalent to f monotone likelihood ratio (MLR) dominates g on $(T, \succeq_d)$. Definition 2 is an extension of MLR dominance to a partially ordered set that only imposes the likelihood ratio inequality

[14] Naturally, one can only compare the actions for types in the mutual support.

on comparable pairs of types. That is, the more evidence relation places no restriction on the relative probability of different kinds of evidence, e.g. a distribution with more evidence can either decrease or increase the relative probability of DNA evidence to witness testimony in a criminal trial. Note that even when a pair of types is incomparable under $\succeq_d$, these types can still influence each other's ROE actions by pooling with some other type that they both dominate in $\succeq_d$. In the previous example, the prosecutor with DNA evidence influences the conviction probability of the prosecutor with witness testimony if both types of prosecutors withhold their evidence in equilibrium.

3.2. The Equivalence Result

Theorem 1. *Let* $f, g \in \Delta T$*,*

$$f \geq_{ME} g \implies f \geq_{MS} g.$$

If f *and* g *have full support then*

$$f \geq_{ME} g \iff f \geq_{MS} g.$$

The result says that if f has more evidence than g, all types obtain a lower action under f than g for any receiver preferences.[15] If both distributions have full support, then the converse also holds: if f does not have more evidence than g, then there will exist a receiver that treats some type strictly more favorably under f than under g.[16] One can restate Theorem 1 as follows: the sender prefers to induce any receiver to hold one belief over another, regardless of his true ex-ante distribution over evidence, if and only if the former has less evidence than the latter.

As an illustration, consider the multidimensional Dye evidence model from Subsection 2.1. Suppose that the distribution over types is generated first by $q \in$

[15] The qualifier that a more skeptical distribution induce lower actions for *any* receiver preferences may be overly demanding in certain applications. For example, an entrepreneur with positive customer reviews will have higher value according to any investor than one with negative customer reviews. Subsection 7.3 develops a definition of more skepticism that need only hold for receiver preferences that adhere to these kind of restrictions. I then characterize this concept in terms of the more evidence relation with respect to a coarser order than $\succeq_d$, i.e. the likelihood ratio inequality is imposed on fewer pairs of types.

[16] The caveat of full support in the equivalence is because the more evidence definition imposes restrictions outside of the supports of f and g and these are not always relevant to the ROE actions. A more complete converse is established in the proof: if $f(t)g(t') < f(t')g(t)$ for some $t \succeq_d t'$ such that $t' \in \text{Supp}(f) \cap \text{Supp}(g)$ then $\exists U^R \in \Upsilon$ such that $\pi_f(t'|U^R) > \pi_g(t'|U^R)$.

$\Delta(2^{\{1,2,...,K\}})$ which determines which pieces of evidence the sender obtains, and then $h_i \in \Delta E^i \setminus \{e^i_\emptyset\}$ independently determines what kind of evidence the sender obtains along each dimension. For example, q determines whether the prosecutor obtains DNA evidence, witness testimony, financial records, or any subset of these, and $\{h_i\}$ determines whether the DNA matches the suspect, whether the witness confirms the alibi, and the degree of financial motive. Consider two distributions over types f and g which are induced by the same h_i, but by different q_f and q_g, i.e. change the probability of obtaining each type of evidence without changing the distribution of its realization. f has more evidence than g if $\frac{q_f(S)}{q_g(S)}$ is increasing in the containment order for $S \subset \{1, 2, ..., K\}$. For example, obtaining DNA evidence *and* witness testimony is relatively more likely under q_f than q_g as compared to obtaining just witness testimony. Note that this comparison puts no restriction on the relative probability of obtaining witness testimony *or* DNA evidence under q_f than q_g. Theorem 1 says that the sender does worse for every type realization when he faces a receiver that holds beliefs f vs. beliefs g.[17]

Using the Equivalence Result It is worth noting that Theorem 1 does not imply obvious welfare comparative statics in the standard verifiable disclosure model. Consider comparing two disclosure games that only differ in the distribution of sender's evidence– either $f \in \Delta T$ or $g \in \Delta T$ where $f \geq_{ME} g$. While the main result says that the sender obtains a lower action for every type realization under f than g, the ex-ante utility of the sender can still be higher under f than g.[18] That is, the prosecutor who is perceived to have more evidence by the jury does worse, however the prosecutor who actually has more evidence ex-ante may do better.

The relevant welfare comparison for Theorem 1 is about comparing different receiver perceptions about the distribution of evidence: fixing an arbitrary actual evidence distribution of the sender; the sender would prefer to be believed to have less evidence. This comparison is the relevant one for signaling interactions before the disclosure game: when comparing two signals, the sender effectively manip-

[17] It is also worth noting that maintaining $\{h_i\}$ between f and g is not necessary for them to be comparable under the more evidence relation. Suppose that f and g are induced by $(\{h^f_i\}, q^f)$ $(\{h^g_i\}, q^g)$ respectively. If $\frac{q_f(S)}{q_g(S)}$ is strictly increasing in the containment order for $S \subset \{1, 2, ..., K\}$, then there exists $\varepsilon > 0$ such that if $\frac{h^f_i(e^i_j)}{h^g_i(e^i_j)} \in (1-\varepsilon, 1+\varepsilon)\ \forall i, j$ then f has more evidence than g.

[18] For example, suppose that the disclosure order is empty. The ROE has full separation and the actions do not depend on the distribution. This means that $\forall f, g \in \Delta T$, $f \geq_{MS} g$, which means that it is simultaneously possible that $\mathbb{E}[v(t)|t \sim f] \geq \mathbb{E}[v(t)|t \sim g]$.

ulates the receiver's belief about his evidence, while holding his true distribution over evidence constant. How do financial disclosures in the current year affect the market's view of non-disclosure in the next year? How does an entrepreneur disclosing early development progress affect investor demands for future progress?

I show the usefulness of Theorem 1 in dynamic disclosure in Section 6. I first develop a simple two period workhorse model of such a dynamic signaling interaction. Corollary 1 details how the characterization of more skepticism is used. The main takeaway is that if period 1 disclosures induce period 2 receiver beliefs that are ordered by the more evidence relation, then the sender must be compensated for inducing the more evidence belief through a higher action in period 1.

While the context of the workhorse model is general, the result is only activated in the context where induced equilibrium beliefs are comparable by the more evidence relation. I take two steps to suggest wide applicability. First I explain how models in Grubb (2011), and Guttman et al. (2014) are special cases of the two period workhorse model and that some of their results use predecessors of Theorem 1 and Corollary 1. Second, I develop two applications. The first extends some insights from repeated disclosure to more general evidence structures while also showing the limitations of others. In particular, early non-disclosure is met with less skepticism when the sender is more patient. The second is a new application concerning the impossibility of pure signaling in dynamic disclosure: if both the sender and the receiver only care about the final action and final type respectively, then Proposition 5 characterizes the evidence structures where the receiver never benefits from early disclosures.

Analysis Road Map The difficult direction in proving Theorem 1 is that more evidence implies more skepticism. One would like to build on the intuition provided for similar results in the Dye model. That is, when the sender omits evidence, i.e. mimics a less dominant type, he does so because he has relatively lower value. Thus conditional on a set of "pooled types", i.e. types who obtain the same action, a more evidence change shifts probability "up the disclosure order" on each of these pooled sets to the types with relatively lower value. Intuitively, this decreases the receiver's best response to each pooled set, i.e. induces more skepticism. However this intuition is incomplete, and the next two sections (road mapped below) provide the necessary analysis to establish Theorem 1.

The simple kind of monotonicity that characterizes pooled sets in the Dye model does not hold in more general evidence environments: it is not generally true that dominated types within a pooled set according to the disclosure order have lower value. In addition, the pooled sets in the ROE can depend on the receiver's beliefs. Thus, solving for the ROE is key to establishing Theorem 1. Accordingly, in Section 4 I characterize pooled sets in terms of the receiver's preferences and his beliefs over the sender's type. I use this characterization to provide a construction of these pooled sets, as well as an explicit expression for the ROE allocation.

However, even given the characterization of pooled sets, existing comparative statics tools do not imply that the value of each pooled set decreases under a more evidence shift. Section 5 establishes this conclusion by introducing an algorithm that constructs pooled sets iteratively based on incentives to mimic. I show that each step of the algorithm decreases the receiver's value. Finally, I briefly discuss how to handle changes in the ROE pooled sets.

4. Equilibrium Characterization

The equivalence between more skepticism and more evidence relies on the structure of the ROE. This section characterizes this structure and provides two ways to find the corresponding equilibrium actions. Since I focus on a single distribution in this section, it is without loss to assume a full support receiver prior belief $h \in \Delta T$.

The following notation for upper and lower contour sets will be useful. For $S \subset T$, let $\boldsymbol{W(S)} \equiv \{t \in T : \exists s \in S,\ s \succeq_d t\}$ be the lower contour set of S according to $\succeq_d$. These are the types that are **worse** than some type in S by $\succeq_d$, or equivalently, the set of types that can be mimicked by some type in S. Similarly, for any subset $S \subset T$, let $\boldsymbol{B(S)} \equiv \{t \in T : \exists s \in S,\ t \succeq_d s\}$ be the upper contour set of S according to $\succeq_d$. This is the set of types that are **better** than some type in S by $\succeq_d$, or equivalently, the set of types that can mimic some type in S. This notation omits the dependence on the disclosure order. When dealing with other ordered sets $(X, \geq)$, I refer to the lower and upper contour set functions as $W_{\geq}$ and $B_{\geq}$ respectively.

4.1. Equilibria as Partitions

Let $\pi : T \to \mathbb{R}$ be some (not necessarily receiver optimal) equilibrium allocation with $\pi(T) = \{\pi_1, ..., \pi_n\}$ as the ordered range of π (i.e. $i < j \implies \pi_i < \pi_j$). Denote the equivalence classes induced by π as $P_i = \{t \in T : \ \pi(t) = \pi_i\}\ \forall i = 1, ..., m$.

I refer to $P = (P_1, ..., P_m)$ as an *equilibrium partition* and each P_i as an equilibrium *pooled set*. In words, P splits the types according to their equilibrium outcomes.

While the allocation obviously pins down the equilibrium partition, an equilibrium partition also pins down the allocation. Because each $t \in P_i$ is assigned the same action π_i, this action must be the receiver's best response to the set as a whole, i.e. $\pi_i = V_h(P_i)$. Saying that $P = (P_1, ..., P_m)$ is an equilibrium partition thereby means that there is an equilibrium which allocates $V_h(P_i)$ to every $t \in P_i$. By the convention above, $V_h(P_i)$ is increasing in i.

In addition, any equilibrium partition must respect the sender's incentives. That is if $t' \succeq_d t''$ then $\pi(t') \geq \pi(t'')$. In terms of the equilibrium partition, this means that $t' \in P_i$ and $t'' \in P_j$ implies that $i \geq j$. I call a partition satisfying this property an *interval partition*. This definition is formalized below in the context of arbitrary pre-orders (which will be useful later).

Definition 3. Let $(X, \geq)$ be a pre-ordered set. An **interval partition** of $(X, \geq)$ is $P = (P_1, ..., P_m)$ such that

$$x, x'' \in P_i,\ x \geq x' \geq x'' \implies x' \in P_i.$$[19]

To summarize, if P is an equilibrium partition, then P is an interval partition and $V_h(P_i)$ is increasing in i. The one remaining feature to characterize equilibrium partitions is the existence of a *pooling strategy* for each P_i, i.e. $\sigma_{|P_i} : P_i \to \Delta P_i$ such that each on path type declaration in P_i induces the same best response from the receiver.[20] Even with this additional condition, unrefined PBE pin down very little about which sets are pooled.[21] However, focusing on the ROE guarantees additional structure on pooled sets.

[19] The term "interval partition" is used because for any interval partition $P = (P_1, ..., P_m)$ of the reals $(\mathbb{R}, \geq)$, each part is an interval, i.e. $P_i = [a, b]$, $\forall i$, for some (extended) real numbers $a \leq b$.

[20] This condition is not pivotal in the analysis and so I defer its characterization to Subsection A.2.

[21] For an example of non-uniqueness consider that $T \equiv \{t_1, t_2, t_3\}$ with $t_3 \succ_d t_2 \succ_d t_1$, U^R is quadratic loss, and $v(t_1) = 0$, $v(t_2) = 10$, and $v(t_3) = -5$. Under any distribution $h \in \Delta T$ such that $h(t_2) = h(t_3)$, the ROE partition is given by $(\{t_1\}, \{t_2, t_3\})$. However, full pooling where all types declare t_1 is also a PBE (with any other declaration assumed to come from t_3). Theorem 1 does not hold for this pooling equilibrium, as the equilibrium action for all types is given by $V_h(T) = 5/2H(\{t_2, t_3\})$ which is strictly increasing in $H(\{t_2, t_3\})$ – a more evidence change.

4.2. The Receiver Optimal Equilibrium Partition

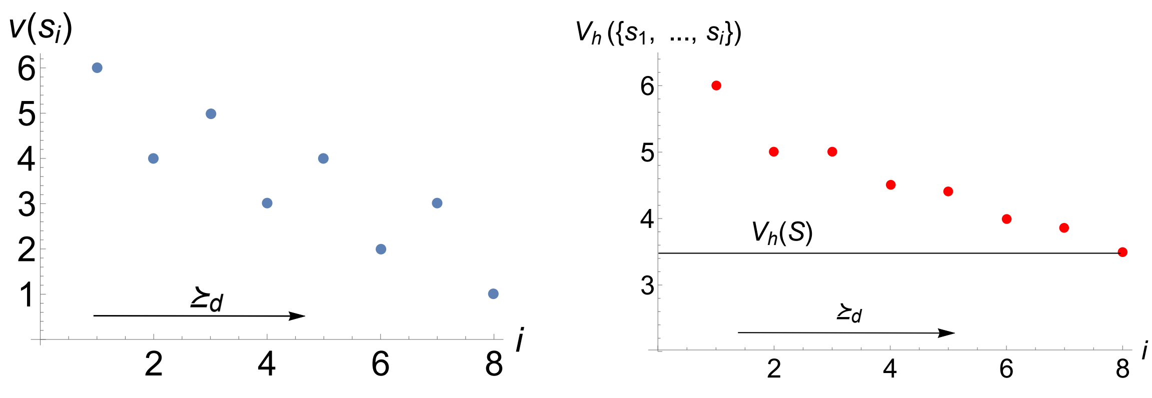
In the ROE, pooled sets have the additional property that they cannot be further "separated". Imagine separating some pooled set P into two parts $\underline{P}$ and $\overline{P}$, where $V_h(\overline{P}) \geq V_h(P) \geq V_h(\underline{P})$. Clearly this separation provides the receiver with better information about the sender's type. Thus in the ROE this kind of separation must be prevented by the sender's incentives. One possibility is that sender types in $\underline{P}$ have the ability to mimic types in $\overline{P}$, i.e. $W(\underline{P}) \cap \overline{P} \neq \emptyset$. This is formalized in the following condition.

Definition 4. The receiver's best response, V_h, is **downward biased** on $(S, \succeq_d)$ if

$$V_h(W(\tilde{S}) \cap S) \geq V_h(S) \; \forall \tilde{S} \subset S. \tag{3}$$

I also refer to sets over which V_h is downward biased as *downward biased sets*.

Example 1. Let $(S, \succeq_d)$ be the complete order described in Subsection 2.1. If V_h is downward biased on $(S, \succeq_d)$, then $\forall i = 1, ..., m$, $V_h(\{s_1, ..., s_i\}) \geq V_h(S)$. A specific example is illustrated in Figure 3.[22] This demonstrates that on a completely ordered set, V_h being downward biased on $(S, \succeq_d)$ is weaker than v being decreasing on $(S, \succeq_d)$. This potential non-monotonicity is the reason that the simple intuition for Theorem 1 described in the analysis roadmap does not go through. △



(a) Best response to types in S

(b) Best response to lower contour sets

Figure 3: V_h is downward biased on $(S, \succeq_d)$

[22] In this case the receiver's utility is quadratic loss, $S \equiv (s_1, ..., s_8)$, and h is the uniform distribution. The left panel shows the receiver's best response to each type and the right panel shows the receiver's best response to lower contour subsets.

Note that by definition, $W(\tilde{S}) \cap S$ cannot mimic its complement in S. Thus, the receiver could separate types in $W(\tilde{S}) \cap S$ from S and give them a lower action and their complement a higher action, while still preserving sender incentive compatibility. If V_h is downward biased on $(S, \succeq_d)$, then this separation is not possible for the receiver as his best response for each lower contour subset is actually higher than that for its complement in S. It turns out that the downward biased condition is both necessary and sufficient for ROE pooled sets as stated in Proposition 1 below.

Proposition 1. *Let P be a partition of T, where $V_h(P_i)$ is increasing in i. P is the unique receiver optimal equilibrium partition if and only if*

$$V_h \text{ is downward biased on } (P_i, \succeq_d)\ \forall i, \text{ and} \tag{4}$$

$$(P_1, ..., P_m) \text{ is an interval partition of } (T, \succeq_d). \tag{5}$$

It is clear that if a partition satisfies the conditions of Proposition 1 the downward biased condition means that it cannot be refined and preserve sender incentive compatibility. However, this does not mean that such a partition is receiver optimal. Alternative equilibrium partitions can also be incomparable with the receiver optimal partition, i.e. neither refinements nor coarsenings.

To get an intuition for the argument for Proposition 1, let P^* be an interval partition satisfying the downward biased condition on each part and let P be some arbitrary alternative equilibrium interval partition. I show that the receiver does better under P^* than under P on each part P_i^*. This is potentially counter-intuitive: because P^* does not necessarily refine P, the receiver assigns types in P_i^* a variety of actions under P while pooling them all at $V_h(P_i^*)$ under P^*. It turns out that because of the downward biased property, the variety in actions under P is tailored to exactly oppose the receiver's preferences.

To see why, let $a_k \equiv V_h(P_k)$ be the allocation under P and note that P_i^* is the union over $Q_k \equiv P_j \cap P_i^*$. That is Q_k refers to the types in P_i^* that get action a_k under P. Recall that by notational convention a_k is strictly increasing in k, so let $a_1 < ... < a_{k'} < V_h(P_i^*) < a_{k'+1} < ... < a_n$. Observe that for each j, $\cup_{k=1}^{j} Q_k$ is a lower contour subset of P_i^* and so $V_h(\cup_{k=1}^{j} Q_k) \geq V_h(P_i^*)$. Similarly $\cup_{k=j}^{n} Q_k$ is an upper contour subset of P_i^* and so $V_h(\cup_{k=j}^{n} Q_k) \leq V_h(P_i^*)$. In words P gives lower actions than P^* to subsets of types for which the receiver actually prefers higher actions and vice versa. Figure 4 illustrates this pattern. The proof of Proposition 1 transforms the alternative allocation by moving the actions down to $V_h(P_i^*)$ for the lower contour subsets (respectively up for the upper contour subsets) in a way that improves the receiver's utility at each stage.[23]

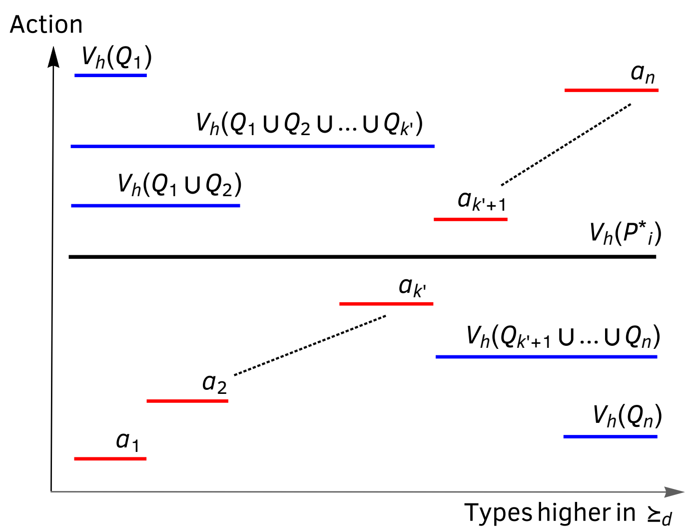


Figure 4: An ROE part P_i^* intersects an alternative partition P.

4.3. Solving for the Receiver Optimal Equilibrium

The previous discussion shows that a partition satisfying the conditions of Proposition 1 is receiver optimal. This subsection constructs such a partition and solves for the ROE allocation.

Lemma 1. *For any set $(S, \succeq_d)$, let $\underline{J} \subset \arg\min_{\tilde{S} \subset S} V_h(W(\tilde{S}) \cap S)$. $\cup_{\hat{S} \in \underline{J}} W(\hat{S}) \cap S$ is a downward biased set. Let $\overline{J} \subset \arg\max_{\tilde{S} \subset S} V_h(B(\tilde{S}) \cap S)$. $\cup_{\hat{S} \in \overline{J}} B(\hat{S}) \cap S$ is a downward biased set.*

Lemma 1 says that V_h is downward biased on any minimal-valued lower contour subset. If there are multiple minima, then V_h is downward biased on any union. The ability to find downward biased sets is useful in finding the ROE partition. Consider applying the above result as follows. Begin with the entire type

[23] Note that this analysis makes no mention of the sender strategies that sustain pooling on each part of the relevant equilibrium partition. This means the argument could be directly interpreted as applying to the optimal mechanism where the receiver has commitment.

set T and use Lemma 1 to find a downward biased P_1. Next remove P_1 and apply Lemma 1 to $T \setminus P_1$ to find another downward biased set P_2. Repeat this process, until the type space is exhausted. This algorithm, which I call *partition into pooled sets*, generates the ROE partition.

ALGORITHM 1: Partition into Pooled Sets

Input: $(T, \succeq_d)$
Output: ROE partition
$i = 1$; $S_1 = T$;
while $S_i \neq \emptyset$ **do**
  $\overline{P}_i = \arg\min_{\tilde{S}_i \subset S_i} V_h(W(\tilde{S}_i) \cap S_i)$;
  $P_i = \cup_{S \in \overline{P}_i} S$;
  $i = i + 1$;
  $S_i = S_{i-1} \setminus P_{i-1}$;
end

Proposition 2. *The output of "Partition into Pooled Sets" $(P_1, ..., P_m)$ is the receiver optimal equilibrium partition with $t \in P_i \implies \pi_h(t|U^R) = V_h(P_i)$.*

The algorithm constructs the ROE partition "bottom-up", i.e. starting with the lowest payoff part. When the minimization is replaced with a maximization, and the W operator is replaced with the B operator, the algorithm constructs the same equilibrium partition "top-down", i.e. starting from the highest payoff part. The next result extends this logic to find an explicit min-max expression for the ROE action for each type.

Theorem 2. *The receiver optimal equilibrium allocation is given by the following expression.*

$$\pi_h(t|U^R) = \min_{\{S_a\colon t \in S_a\}} \max_{\{S_b\colon t \in S_b\}} V_h(W(S_a) \cap B(S_b)). \tag{6}$$

The interpretation of Theorem 2 is that the pooled set for a given type t results from the combination of two forces. First, type t chooses some set of dominated types to pool with in order to increase the receiver best response. Second, the types dominating this chosen set, will pool with t if it improves their value. This latter process serves to lower the best response to t as these more dominant types will only pool with t if they have relatively lower value. Thus the min-max in

(6) comes from (i) types that dominate t pooling with t to minimize his action (because it improves their own) and (ii) t pooling with types he dominates in order to maximize his action.

This section characterized ROE pooled sets. The next section uses these characterization results to show that the ROE actions decrease under a more evidence shift, i.e. to provide intuition for why more evidence implies more skepticism.

5. Why More Evidence implies More Skepticism

In this section I assume all distributions have full support.[24] Consider two distributions $f, g \in \Delta T$, such that $f \geq_{ME} g$. In general, the ROE partitions under f and g can be different. However, I will first show why more evidence implies more skepticism under the assumption that the ROE partitions are the same.[25] At the end of this section I will discuss the intuition for adapting this argument to when the ROE partition changes between f and g.

If the pooled sets are the same under f and g, establishing that $f \geq_{MS} g$ comes down to proving that a more evidence shift decreases the value of each pooled set. In light of Proposition 1, I prove the following result.

Proposition 3. *Consider* $(S, \succeq_d)$ *and* $f \in \Delta S$. $V_f(S) \leq V_g(S)$ $\forall g \in \Delta S$ *such that* $f \geq_{ME} g$ *with respect to* $(S, \succeq_d)$ *if and only if* V_f *is downward biased on* $(S, \succeq_d)$.

The result says that the condition that characterizes pooled sets in the ROE also characterizes monotone comparative statics (MCS) under any more evidence shift. One direction is relatively straightforward. If V_h is not downward biased on $(S, \succeq_d)$, there is a lower contour subset with lower value than S as a whole. Moving probability from this subset to its complement is a more evidence shift and increases the value of S.

The other direction in Proposition 3 is more complicated. One would like to use the following well known comparative statics result: the expected value of a decreasing function is lower under a monotone likelihood ratio shift. This result appeared in Topkis (1976) and is formally reproduced below in terms of receiver best responses.

[24] Subsection B.6 shows how to use these results to prove Theorem 1 which does not assume $f, g \in \Delta T$ have full support.

[25] For example, consider $\tilde{f} \geq_{ME} g$ and let $f(t) \equiv \alpha \tilde{f}(t) + (1-\alpha)g(t)\,; \forall t$, for some small $\alpha \in (0, 1]$. Generically, the ROE partition will be the same under f and g.

Fact 1. *Let two distributions $f, g \in \Delta S$. If $\forall t, t' \in S$, $f(t)g(t') > f(t')g(t) \implies v(t) \leq v(t')$, then $V_f(S) \leq V_g(S)$.*

If the disclosure order were complete, and the receiver's best response were decreasing in the disclosure order on any pooled set, the above fact would yield Proposition 3. The problem is that the disclosure order is not complete, and even if it were, as Figure 3 illustrates, no such monotonicity property holds on the ROE pooled sets. The next section instead uses the above fact iteratively to establish MCS for downward biased sets.

5.1. Iteratively Pooling Subsets

I prove Proposition 3 by introducing an algorithm that iteratively pools subsets of types based on incentives to mimic. Fix some pooled set S. Roughly, at each stage a subset of types in S pools with another if the former has higher value under a distribution $f \in \Delta S$ and "adjacently" dominates the latter set. I will show that when S is a downward biased set, i.e. it would be pooled in the ROE, this process eventually pools all types in S.

For any two distributions $f, g \in \Delta S$, define the induced $f - g$ likelihood ratio pre-order, $\geq_{f/g}$, as $\forall s', s'' \in S \; f(s')g(s'') \geq f(s'')g(s') \implies s' \geq_{f/g} s''$. $(S, \geq_{f/g})$ is a completely ordered set of types, in which higher types are relatively more likely under f than under g. For convenience, refine $\geq_{f/g}$ to a complete order by breaking ties according to the disclosure order and arbitrarily otherwise.

The formal description of the algorithm is presented below. However its implementation and use in the proof of Proposition 3 is best illustrated by the proceeding example.

Description of the Algorithm The algorithm begins with the complete interval partition of $(S, \geq_{f/g})$, $Q^1 = (\{t_1\}, \{t_2\}, ..., \{t_m\})$. Beginning with t_1, the algorithm repeatedly forms the largest sequence of elements such that $v(t_j)$ is decreasing in j. That is, the first sequence is $\{t_1, t_2, ..., t_{I_1}\}$ such that $v(t_1) \geq ... \geq v(t_{I_1})$ and $v(t_{I_1}) < v(t_{I_1+1})$, the second sequence is $\{t_{I_1+1}, ..., t_{I_2}\}$ such that $v(t_{I_1+1}) \geq ... \geq v(t_{I_2})$ and $v(t_{I_2}) < v(t_{I_2+1})$, and so on until all types in S are exhausted. Next, a coarser interval partition Q^2 is formed by pooling all the elements of each decreasing sequence into an associated single part with value determined by $V_f(\cdot)$. That is, $Q_1^2 \equiv \{t_1, ..., t_{I_1}\}$, $Q_2^2 = \{t_{I_1+1}, ..., t_{I_2}\}$, and so on. This process is repeated: at each stage, Q^i is coarsened into Q^{i+1} where each part of Q^{i+1} pools a consecutive sequence of Q^i_j over which $V_f(Q^i_j)$ is decreasing in j. The algorithm concludes at stage T defined by $Q^T = Q^{T+1}$, i.e. where $V_f(Q^T_i)$ is strictly increasing in i.

Example 2. Recall the example in Figure 3 in which V_f is downward biased on $(S, \succeq_d)$. Specifically, the disclosure order is complete on $S = (s_1, ..., s_8)$, f is the uniform distribution on S, and $(v(s_1), ..., v(s_8)) = (6, 4, 5, 3, 4, 2, 3, 1)$. Let $g \in \Delta S$ such that $f \geq_{ME} g$. Since the disclosure order is complete, $\geq_{f/g} = \succeq_d$. The goal is to show that $V_f(S) \leq V_g(S)$ establishing MCS under any less evidence shift in this case. However as v is not decreasing, one cannot directly apply Fact 1. Instead one can pool types iteratively such that at each stage the value of the currently pooled subset is lower under the more evidence distribution.

Since $v(s_1) > v(s_2)$, s_2 will pool with s_1 regardless of the distribution. The algorithm pools $\{s_1, s_2\}$ into a "single type" with value given by $V_f(\{s_1, s_2\})$. Similar logic also pools $\{s_3, s_4\}$, $\{s_5, s_6\}$, and $\{s_7, s_8\}$. The result is the partition P_2 illustrated in the left panel of Figure 5. Notice that since $v(s)$ is decreasing on each P^2_i, Fact 1 implies that $V_f(P^2_i) \leq V_g(P^2_i),\ \forall i = 1, ..., 4$.

Next, note that since $V_f(P_i)$ is decreasing in i, these sets will all pool together in the ROE. Indeed the second step of the algorithm pools $\{P^2_1, ...P^2_4\}$ into a single pooled set with value $V_f(S)$. The result is a coarser (trivial) partition P^3 illustrated in the right panel of Figure 5. Putting this altogether gives

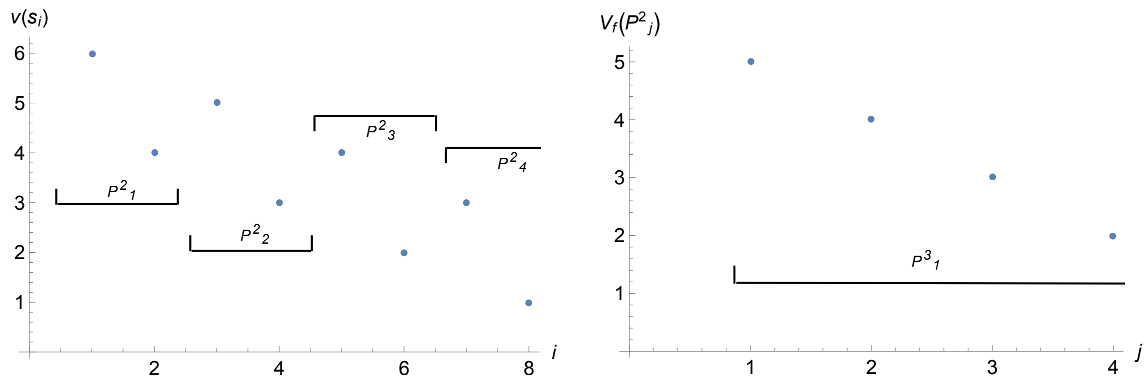
$$V_f(S) = \sum_{i=1}^{4} V_f(P^2_i)F(P^2_i) \leq \sum_{i=1}^{4} V_f(P^2_i)G(P^2_i) \leq \sum_{i=1}^{4} V_g(P^2_i)G(P^2_i) = V_g(S).$$

As before, since $V_f(P^2_i)$ is decreasing in i, Fact 1 implies the first inequality, and $V_f(P^2_i) \leq V_g(P^2_i)$ – gleaned at the first stage – gives the second inequality. The downward biased property ensures that the algorithm always concludes at the trivial partition.

△

5.2. Changes in the Equilibrium Partition

The preceding analysis is only sufficient to show that more evidence implies lower equilibrium actions in the case when the ROE partition is constant across distributions f and g. This section provides some intuition for how this argument



(a) Stage 1 (b) Stage 2

Figure 5: Applying the Algorithm to a Downward Biased Set

extends to cases in which the ROE partition changes when the sender has more evidence.

Consider that $f \geq_{ME} g$. For simplicity, let $P^g = (P_1^g, P_2^g)$ have two elements, while $P^f = (P_1^f)$, has only one. For $\alpha \in [0,1]$, define the combination distribution, $h_\alpha \equiv \alpha f + (1-\alpha) g$ with corresponding ROE partition $P^\alpha \equiv (P_1^\alpha, ..., P_{m_\alpha}^\alpha)$. First, since the equilibrium action is increasing in the disclosure order, $V_g(P_1^g) < V_g(P_2^g)$, and second, since the receiver's best response is downward biased on each part, $V_f(P_1^g) \geq V_f(P_2^g)$.

Because the receiver's best response to these subsets is continuous in α, there must exist some α^* above which the ROE partition changes from P^g to P^f and $V_{h_{\alpha^*}}(P_1^g) = V_{h_{\alpha^*}}(P_2^g)$. For simplicity, suppose that this is the only change in the ROE partition as α increases from 0 to 1.

Notice that for $\alpha > \alpha'$ $h_\alpha \geq_{ME} h_{\alpha'}$, and so Proposition 3 completes the argument in this case. The idea is that each equilibrium part – P_1^g and P_2^g – decreases in value as α increases, until equalizing at $\alpha = \alpha^*$. For $\alpha > \alpha^*$, all types pool together and so again by Proposition 3, the value of $P_f^1 = S$ decreases in α. Figure 6 illustrates the ROE actions as a function of α.

6. Dynamic Disclosure

This section shows how Theorem 1 can be used. First I develop a workhorse dynamic disclosure model. I establish Corollary 1 below which relates the more skepticism characterization to the signaling preferences of the sender in period 1. I

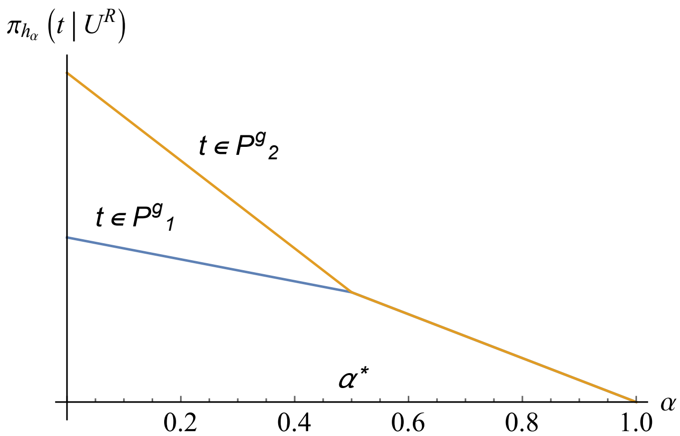


Figure 6: ROE Actions under Changes in the ROE Partition

then discuss how the literature has used previous incarnations of more skepticism to analyze specific versions of this workhorse model. Finally Subsection 6.2 and Subsection 6.3 introduce two applications that use the more skepticism characterization, and Corollary 1 to develop new insights.

6.1. A Simple Dynamic Disclosure Model

First Period In the first period, the sender obtains private information $s \in S$ where S is some finite set. The distribution of s is given by $h_1 \in \Delta S$. The message availability is potentially limited in period 1 according to the disclosure preorder $\succeq_d^1$ on S.[26] That is, the message space is S, and the sender with type s can choose to send any message $\{s' \in S : s \succeq_d^1 s'\}$. The receiver observes the message s' and then chooses an action $a_1 \in \mathbb{R}$.

Second Period In the second period, the sender obtains evidence $t \in T$ where $(T, \succeq_d^2)$ is a disclosure ordered type space. The distribution of t given private information s in period 1 is given by $h_2(\cdot|s) \in \Delta T$. The sender and receiver then engage in the disclosure game described in Section 2 with the receiver choosing an action $a_2 \in \mathbb{R}$.

Preferences The receiver and sender are assumed to have time separable preferences. The receiver's preferences only depend on the current type of the sender,

[26] Since different period 1 types may have different distributions in period 2, and thereby different preferences about period 2 allocations, removing cheap talk is not without loss.

i.e. his total utility is given by $U_1^R(a_1, s) + U_2^R(a_2, t)$ where U_1^R and U_2^R are concave in the action for any sender type. Define the corresponding best response functions v_i and V_h^i in period i. The sender wants higher actions in both period 1 and period 2 and his utility is given by $ra_1 + a_2$ where $r \geq 0$ is a positive weight.

Strategies and equilibrium A sender strategy in period 1 is a distribution over $\{s' \in S : s \succeq_d^1 s'\}$ for each realization of $s \in S$ – $\sigma_s(s')$. Consider such a period 1 strategy, and define $h_{s'} \in \Delta T$ as the interim updated belief of the receiver about the period 2 evidence having observed s' in period 1 but before communicating in period 2. I restrict the sender to follow the receiver optimal equilibrium strategy in period 2 given prior $h_{s'}$, that is the second period actions are determined by $\pi_{h_{s'}}(t|U_2^R)$. This could be justified by a stronger refinement that the sender and receiver follow truth leaning strategies as in Hart et al. (2017). Alternatively, the receiver could equivalently be assumed to have commitment power in period 2, as the ROE and commitment allocations are equivalent. Let $a(s') \in \mathbb{R}$ be the WLOG receiver's pure period 1 strategy. I focus on PBE that satisfy the additional ROE refinement in period 2.

The first period communication can represent a wide variety of signaling applications, for example it could be another disclosure game a la Section 2 or represent cheap talk communication by letting $\succeq_d^1$ be the preorder where all types are in one equivalence class. In the next subsection, I explain how repeated disclosure with uncertainty about the evidence distribution a la Grubb (2011), or dynamic disclosure where the sender accumulates evidence over time a la Guttman et al. (2014) can be seen as special cases. Now I present the main useful tool that Theorem 1 produces in this dynamic model.

Corollary 1. *Consider a perfect bayesian equilibrium of the above game with σ and a as the sender's and receiver's period 1 strategies. Let $s', s'' \in \{\tilde{s} \in S : s \succeq_d^1 \tilde{s}\}$ with $h_{s'} \geq_{ME} h_{s''}$. If $\sigma_s(s') > 0$, then $a(s') \geq a(s'')$.*

Moreover, suppose $h_{s'}$ and $h_{s''}$ induce different period 2 actions, i.e. $\exists t \in \mathrm{Supp}(h_{s'}) \cap \mathrm{Supp}(h_{s''}) \cap \mathrm{Supp}(h_2(\cdot|s))$: $\pi_{h_{s'}}(t|U_2^R) \neq \pi_{h_{s''}}(t|U_2^R)$. If $\sigma_s(s') > 0$, then $ra(s') > ra(s'')$.

Proof. If $h_{s'} \geq_{ME} h_{s''}$ then $h_{s'} \geq_{MS} h_{s''}$ by Theorem 1. Thus the expected utility of the sender in period 2 following s' is weakly (strictly) less than s'' (if the caveat in the second paragraph holds). Thus if $a(s') < a(s'')(a(s') \leq a(s''))$ then s'' would be strictly preferred to s' and $\sigma_s(s') = 0$. *Q.E.D.*

The corollary says that if a first period disclosure induces a receiver belief with more evidence than some alternative disclosure, it can only be selected if it induces a better first period action. If only final decisions matter for the sender, i.e. $r = 0$, then Corollary 1 immediately implies that first period disclosures cannot induce beliefs ranked according to the more evidence order, unless these induce the same period 2 allocation.

The short proof above shows the central role of the equivalence between more skepticism and more evidence. Because the model does not impose restrictions on the distribution of period 2 evidence given period 1 evidence – h_2 – The MS and thereby the ME order is the "right" notion: the more skepticism order identifies the ex-ante sender preference over receiver beliefs that is common across *all* period 2 evidence distributions.

6.1.1. More Evidence and More Skepticism in the Literature

Grubb (2011) and Guttman et al. (2014) both study dynamic disclosure games of the form above. They establish that specific comparisons in the evidence distribution within the Dye model induce more skepticism. These papers then use an analog of Corollary 1 above for such changes in the evidence distribution to derive various predictions.

Recall the Dye evidence model from Subsection 2.1. Consider two distributions $f, g \in \Delta\{t_\emptyset, t_1, ..., t_{n-1}\}$. $f \geq_{ME} g \iff$

$$f(t_\emptyset)g(t_i) \leq f(t_i)g(t_\emptyset) \quad \forall i = 1, ..., n-1 \tag{7}$$

That is, a more evidence change increases the probability that the sender has all evidence types relative to the no evidence type, and imposes no comparisons on the relative likelihood between evidence types.

Jung and Kwon (1988) parameterize the distribution by setting $p \equiv Pr(\{t_1, ..., t_{n-1}\})$ as the probability of getting evidence and $h \in \Delta\{t_1, ..., t_n\}$ as the conditional distribution over evidence. Let the distribution over types be $h_p \in \Delta T$. They consider increasing p while holding h constant and find that the non-disclosure action ($\pi_{h_p}(t_\emptyset|U^R)$ in my terminology) decreases. In this case, the conditions in (7) reduce to $\frac{1-p_2}{1-p_1} \leq \frac{p_2}{p_1}$, which holds for $p_2 > p_1$.

Corollary 2. *In the Dye Model, if $p_2 > p_1$, then $h_{p_2} \geq_{ME} h_{p_1}$, and $h_{p_2} \geq_{MS} h_{p_1}$.*

This result is well known and has been used in numerous signaling contexts. For example, Grubb (2011) studies a repeated Dye disclosure model with uncertainty about p. In the framework of the previous section, the first and second period communication are both Dye disclosure games, the first period private information s consists of disclosable Dye evidence and information about whether p is high or low. While the second period evidence is independent of the first period evidence, the parameter p is persistent. The author uses Corollary 2 to show that the sender wants to develop a reputation for having a low p, i.e. a low probability of having evidence. This fact combined with an analog to Corollary 1 implies that firms will withhold favorable period 1 evidence in order to signal that they have a lower probability of getting evidence. This in turn increases the action for non-disclosure relative to the static Dye disclosure game.

Grubb (2011) also derives that senders with a low probability of obtaining evidence will have a greater reputational signaling incentive, and so senders with low value evidence in period 1 can effectively signal low p through disclosing this unfavorable evidence. In Subsection 6.2, I use Theorem 1 to show that while the first prediction is robust to considering more complicated evidence structures, the latter conclusion is limited to the Dye evidence model.

Guttman et al. (2014) find that a different change in the prior distribution induces more skepticism in the Dye model. Their change involves augmenting the Dye model with additional evidence types. For any two nested subsets of evidence types $S \subset S' \subset \{t_1, ..., t_{n-1}\}$, they find that the non-disclosure action is lower when the distribution is conditioned on the larger set S'. For an original prior $h \in \Delta T$, define the restricted prior h_S by

$$h_S(t) \equiv \begin{cases} \frac{h(t)}{H(S \cup \{t_\emptyset\})} & \text{if } t \in S \cup \{t_\emptyset\} \\ 0 & \text{otherwise} \end{cases}.$$

To see that $h_{S'} \geq_{ME} h_S$ note that (7) is an equality for $t_i \in S$, and a strict inequality for $t_i \in S' \setminus S$.

Corollary 3. *In the Dye Model, if $S \subset S'$, then $h_{S'} \geq_{ME} h_S$, and $h_{S'} \geq_{MS} h_S$.*

Guttman et al. (2014) term this the "generalized minimum principle", and use it in a two period model where the sender obtains and discloses up to two pieces of evidence gradually. This model also fits in the framework of the workhorse dynamic model: (i) both period 1 and period 2 are disclosure games with the same multidimensional Dye evidence structure with $k = 2$ from Subsection 2.1, and (ii) the distribution of period 2 evidence t depends on period 1 evidence s so that the sender can only gain evidence over time. The authors show that disclosing a single piece of evidence in period 2 is treated more favorably than the same disclosure in period 1. That is, the price for late disclosures is higher then that for early disclosures. Conditional on having disclosed one piece of evidence the remaining uncertainty is about the single remaining piece and therefore operates as in the Dye model. Because he has a better fall-back option, the sender who disclosed in period 1 conceals more realizations of the alternative evidence than the sender who only acquires the disclosed piece in period 2. This means that the induced beliefs in period 2 between early and late disclosures are comparable as in Corollary 3, and thereby the more evidence and more skepticism order.

The next two sections study two new applications of the simple dynamic model introduced in Subsection 6.1. Both applications use Corollary 1 to establish new conclusions. The first application is the most basic multidimensional extension of Grubb (2011). I identify which conclusions are robust to multidimensional evidence structures. The second application addresses a novel question which necessitates the consideration of multidimensional evidence structures: when can the sender make early disclosures in a pure signaling context to improve communication?

6.2. Repeated Disclosure

Consider a firm disclosing earnings guidance over multiple years. There is often public uncertainty about the availability of such verifiable earnings information. Statically, this leads to strategic withholding, as in the Dye evidence model; dynamically this creates incentives to shift shareholders' views about the availability of verifiable information. Grubb (2011) studies a 2 period Dye model in which there is persistent heterogeneity in the probability that the sender obtains evidence. One main conclusion is that the non-disclosure valuation increases in period 1. Another main conclusion is that firms that attain verifiable evidence with high probability will be able to credibly signal this in order to obtain higher non-disclosure prices in future periods. I show that the first conclusion is robust to more complicated evidence structures while the second conclusion is not.

I extend this framework to consider multidimensional evidence. To facilitate a comparison, I assume that the sender still obtains Dye evidence in the first period, but has an arbitrary evidence structure in the second period. I use a version of the model in Subsection 6.1 specified as follows. First period private information is Dye evidence drawn from a distribution dependent on the *type* of the sender, which is H with probability $q \in (0, 1)$ and L with complementary probability. The sender only observes the realized evidence. That is, $S = \{s_\emptyset, s_1, ..., s_{n-1}\}$ with $\succeq_d^1$ specified as usual in the Dye evidence model.[27] The probability distribution of period 1 evidence is given by

$$h_1(s) \equiv \begin{cases} q(1-p_H) + (1-q)(1-p_L) & s = s_\emptyset \\ (qp_H + (1-q)p_L)\, h(s_i) & s = s_i \end{cases},$$

with $1 > p_H > p_L > 0$ and $h \in \Delta\{s_1, ..., s_{n-1}\}$ with full support. That is, the H distribution has more evidence than the L distribution. This relationship continues in period 2, when evidence is drawn from some $(T, \succeq_d)$. Let $f_H, f_L \in \Delta T$ both with full support such that $f_H \geq_{ME} f_L$. The probability distribution over period 2 evidence t given period 1 evidence s is specified as follows:

$$h_2(t|s) = \begin{cases} \frac{p_H q f_H(t) + p_L(1-q) f_L(t)}{p_H q + p_L(1-q)} & s \in \{s_1, ..., s_{n-1}\} \\ \frac{(1-p_H) q f_H(t) + (1-p_L)(1-q) f_L(t)}{(1-p_H) q + (1-p_L)(1-q)} & s = s_\emptyset \end{cases}.$$

That is, period 1 evidence only affects the period 2 distribution of evidence in so far as it affects the sender's Bayesian update about the his type. Recall that the sender's utility is given by $ra_1 + a_2$ where a_i is the action chosen by the receiver in period i. The receiver has arbitrary values for period 1 and period 2 evidence types with three caveats: (i) without loss of generality, $v_1(s_i)$ is weakly increasing in i, (ii) there is some pooling in period 1, i.e. $v_1(s_1) < v_1(s_\emptyset)$, and (iii) there is some pooling in period 2, i.e. $\exists t \succeq_d t'$ such that $v_2(t) < v_2(t')$.

Proposition 4. *Consider an equilibrium of the above dynamic disclosure game under r, and let $a_1(s_\emptyset)$ be the period 1 non-disclosure action. if $r' \leq r$, there exists an equilibrium under r' with associated period 1 non-disclosure action $a_1'(s_\emptyset)$ such that $a_1'(s_\emptyset) \geq a_1(s_\emptyset)$.*

The result says that as the sender values period 1 less, the non-disclosure action in period 1 increases. To interpret the result, note that in static disclosure ($r \to \infty$),

[27] Refer to Subsection 2.1 for a formal description.

a non-disclosure comes from (i) a firm who does not have verifiable earnings information, or (ii) a firm who withholds verifiable but negative earnings. In repeated disclosure (r finite) the non-disclosure could also originate from (iii) a firm with positive earnings information, but who seeks to face a less skeptical market in the future, i.e. one that believes the firm to have less evidence. As the firm values the future more (r increases) point (iii) becomes more impactful relative to points (i) and (ii).

To see the argument, consider a candidate period 1 equilibrium strategy for the sender. The receiver's belief that the sender is the H type following a period 1 disclosure, i.e. a message in $\{s_1, ..., s_{n-1}\}$, is given by $q_e \equiv \frac{p_H q}{p_H q + p_L(1-q)}$. This probability is larger than the receiver's update following non-disclosure, i.e. following the $s_\emptyset$ message.[28] This means that, by Theorem 1, the sender obtains a higher period 2 expected utility from non-disclosure as compared with disclosure. The interpretation is that by not disclosing, the sender generates a reputation that makes the receiver less skeptical in period 2. Thus, by Corollary 1, if the sender discloses evidence s_i in period 1, $v_1(s_i) \geq a_1(s_\emptyset)$, i.e. the sender trades a lower expected action in period 2 for a higher action in period 1. As r decreases, this tradeoff becomes less favorable and the sender will eventually withhold s_i. Since $v_1(s_i) \geq a_1(s_\emptyset)$, this withholding increases the period 1 non-disclosure action.

Grubb (2011) describes an additional interesting prediction in repeated disclosure. That paper assumes Dye evidence in period 2, and that the sender knows whether the distribution over evidence is the H or L type in addition to the evidence realization in each period. It is then possible for L type senders with unfavorable period 1 evidence to disclose in order to credibly signal their type. A key ingredient is that the H type senders will not make such a disclosure because they value reputation less than L types. The reason is that, in the Dye evidence model, the receiver's prior belief only affects the non-disclosure action. The H distribution sender discloses with higher probability than the L distribution sender, and therefore values reputation less. However this feature is specific to the Dye evidence model in period 2, and specifically the existence of a single pooled set. In this case the probability difference over this pooled set between the H and L distributions determines the signaling incentive. If instead the sender's evidence is multidimensional in period 2, there can naturally be more than one pooled set, and the direction of the signaling incentive is ambiguous. In Appendix E, I provide an

[28] The update following non-disclosure is always lower than the prior probability of H types – q.

example in which H types value a reputation for having less evidence more than L types. This precludes the kind of signaling highlighted above.

6.3. Dynamic Evidence Arrival

Consider an entrepreneur strategically disclosing his progress to a venture capitalist. Naturally this progress happens gradually; first, perhaps the entrepreneur attempts to develop a prototype, and only after can he potentially run a performance test. This presents the investor with the opportunity to speak with the entrepreneur at some intermediate stage when potentially not all the evidence has arrived. Could the investor benefit from these additional communications or should he just wait until making his investment to consult with the entrepreneur? Put differently, can there can exist incentive compatible informative disclosure in more than one period?

The depth in this question is particular to multidimensional evidence structures; if the entrepreneur could potentially obtain only a single piece of evidence as in the Dye model, any early signaling would basically end the game. Thus, current comparative statics results centered around the Dye model are ill-equipped to provide an answer.

Below I specify a version of the two period model in Subsection 6.1 to address this question. Both periods are a disclosure game as in Section 2 with the same evidence space and messaging technology given by $(T, \succeq_d)$. The distribution of evidence $s \in T$ in period 1 is given by $h_1 \in \Delta T$ which is assumed to have full support. The probability of evidence $t \in \Delta T$ in period 2 given possession of s in period 1 is given by

$$h_2(t|s) = \begin{cases} \frac{h(t)}{H(B(s))} & \text{if } t \in B(s) \\ 0 & \text{otherwise} \end{cases},$$

for some $h \in \Delta T$ with full support. This implies that possessing more evidence in period 1 makes one more likely to have more evidence in period 2 in the sense of Definition 2. Note that for $t \not\succeq_d s$, the probability of realizing t after s is zero, i.e. the sender does not lose evidence over time. In addition, the distribution of period 2 evidence depends on period 1 evidence only through its upper contour subset. That is, if two types, t, t', both have positive probability in period 2 after two different period 1 realizations, then the relative probability of t to t' will be the same after both period 1 realizations. Note that evidence is not "time-stamped", i.e. the sender cannot credibly convey in period 2 when any disclosed evidence arrived. The only way to credibly convey that evidence arrived in period 1 is to disclose it in period 1.

In order to focus on the pure signaling question I assume $r = 0$, i.e. the sender does not value the first period action and his utility is simply a_2. The receiver also only values the action in the second period and the final evidence realization according to $U_2^R(a_2, t)$.

For a given equilibrium of this dynamic disclosure game, denote $\tilde{\pi} : T \to \Delta\mathbb{R}$ as the distribution of actions as a function of the sender's period 2 type.[29,30] I say that the receiver *benefits from early inspections* if his expected utility in some equilibrium of the dynamic disclosure game is higher than that if the receiver were to only communicate with the sender in period 2.[31] The receiver benefits from early inspections if there exists an equilibrium allocation $\tilde{\pi} : T \to \Delta A$ that is non-degenerate for some $t \in T$, thereby reflecting decision relevant information obtained by the receiver in period 1. Since the receiver is best responding and has strictly concave utility over actions, this non-degeneracy must make him better off. I next introduce the pivotal feature of a disclosure order that will determine whether the receiver can benefit from early inspections.

6.3.1. The Unique Evidence Path Property

Definition 5. A disclosure-ordered type space $(T, \succeq_d)$ has the unique evidence path property (UEPP) if $\forall t, t', t'' \in T$, $t \succeq_d t'$ and $t \succeq_d t''$ implies that either $t' \succeq_d t''$ or $t'' \succeq_d t'$.

Another way to describe the UEPP is that for any type t, $W(t)$ is completely ordered. In this sense, the UEPP says there is a unique "path" in the disclosure order to each type.

One interpretation of the UEPP is that the evidence is gathered through a sequential process of investigations uniquely determined by the realization of the previous one. Another interpretation is that the revelation of evidence also reveals the investigation that led to that evidence. For example a prototype can only be

[29] As this is the only payoff relevant information for both parties, I exclude any description of how $\tilde{\pi} : T \to \Delta A$ depends on period 1 disclosures.

[30] The receiver does not randomize over actions in period 2, rather the potential randomness in $\tilde{\pi}$ arises due to different period 1 disclosures leading to the same period 2 disclosure.

[31] Let the ex-ante beliefs over period 2 types be denoted by $\tilde{h} \in \Delta T$, i.e. $\tilde{h}(t) = \sum_{s \in T} h_1(s) h_2(t|s)$. If the receiver spoke to the sender only in period 2 the allocation $\pi_{\tilde{h}}(t|U^R)$ is given by Theorem 2.

tested if it has been successfully developed. Therefore, revealing a successful performance test would also reveal which successful prototype was developed. In the context of criminal investigations, an alibi can only be reported if the suspect has first been identified. If instead different pieces of evidence can be collected independently, then the UEPP will not be satisfied.

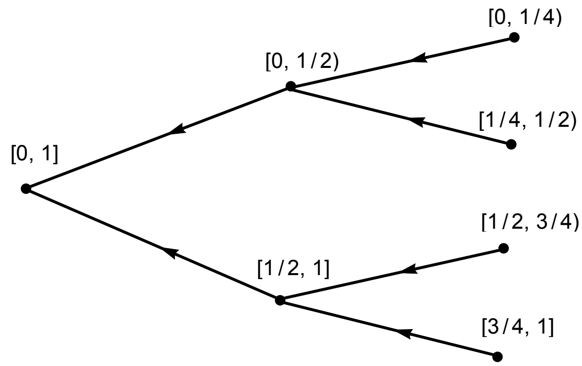
While the Dye model of Subsection 2.1 clearly satisfies the UEPP, the focus in this section is on more complicated evidence structures. Figure 7 illustrates multi-dimensional disclosure models that do and do not satisfy the UEPP. The left panel displays an extended version of the vagueness model that satisfies the UEPP. Any vagueness model in which the evidence space is composed of the parts of a number of successively finer partitions of some set will satisfy the UEPP. The right panel displays an example in which the sender can obtain up to two successes or failures and report them selectively. This example does not satisfy the UEPP because a success and a failure – $\{0, 1\}$, could have originated from a failure – 0, or a success – 1, in period 1.

Proposition 5. *If $(T, \succeq_d)$ satisfies the UEPP, then the only equilibrium allocation is degenerate and the receiver does not benefit from early inspections. If $(T, \succeq_d)$ does not satisfy the UEPP, then there exists h_1, h_2, and U^R such that the receiver benefits from early inspections.*

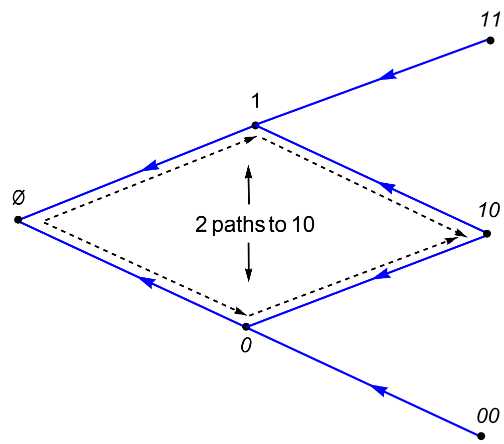
The motivating example of this section, where a developer first tries several ideas for prototypes, and then is potentially able to run a performance test on one that is successfully developed, satisfies the UEPP. Thus Proposition 5 says that the investor would have no benefit from inspecting the entrepreneur's early progress.

Before describing the intuition, it is worth noting that if it could be elicited, early disclosure would be valuable under the UEPP. The result derives instead from the fact that such early disclosures are not incentive compatible for the sender.

The key to the above result is the second part of Corollary 1 specified to the case in which the sender does not value the period 1 action, i.e. $r = 0$. Towards a contradiction, consider two possible disclosures in period 1, $s, s' \in T$, with associated induced period 2 receiver beliefs, $h_s, h_{s'} \in \Delta T$, that induce different actions in period 2. If $h_s \geq_{ME} h_{s'}$, then since there is no compensating first period action, s will never be disclosed by the sender in period 1. Roughly, the argument for Proposition 5 shows that any two such period 1 disclosures will induce period 2 receiver beliefs that are ranked by the more evidence relation "from the perspective of some



(a) Extended Vagueness satisfies the UEPP



(b) Multidimensional Dye violates the UEPP

Figure 7:

type". Consequently the action profile in period 2 must be the same following any on path period 1 disclosure.

An intuition for why the absence of the UEPP can lead to informative early communication is as follows. The UEPP implies that any two period 1 types that cannot mimic each other lead to period 2 type realizations that also cannot mimic each other. In this sense, any potential for separation in period 1 is maintained in period 2. Without this property, two sender type realizations who are not able to mimic each other in period 1 could give rise to the same evidence type in period 2 thereby losing this potential for separation. In this case, there is clearly scope for informative period 1 communication. An example with informative dynamic signaling under a violation of the UEPP is presented in Appendix F.

7. Extensions

This section analyzes key extensions under which the main results apply. I show how two kinds of uncertainty over the sender's preferences can be incorporated in the baseline model through alterations to the disclosure order. First I consider uncertainty over whether the sender has an upward or downward bias. Within this framework I show how the sender would want to signal that his preference is the opposite of his own, and detail when the ROE will reveal the sender's preference to the receiver. Next I incorporate a probability that the sender is unbiased by showing that this is equivalent to the game with the same probability of honest types. Finally, I discuss how to relax the definition of the more evidence relation to characterize a notion of more skepticism in which the modeler has some information about the receiver's preferences.

7.1. Senders with Unknown Bias

There are many reasons why the sender may not always prefer higher actions. The sender may be biased but in an unknown direction. For example, a police officer who is usually partial to conviction may want to exonerate a suspect with whom he has a relationship, or a lobbyist may represent an anonymous company. When will the sender credibly convey the direction of his bias to the receiver? Which prior beliefs about his preferences does the sender want to induce in the receiver?

The sender prefers higher actions (type H) with utility $U^S(a) = a$, with probability p, and prefers lower actions (type L) with utility $U^S(a) = -a$, with probability $1-p$.[32] In addition to private information about his preferences, the sender obtains disclosable evidence from $(T', \succeq_d')$. The total type space is given by $T \equiv T' \times \{H, L\}$. The distribution over evidence can depend on the preference of the sender: let $f^H, f^L \in \Delta T'$ be the marginal distributions for H and L preferences respectively. Denote the unconditional distribution over types, $h_p \in \Delta T$. The receiver's preferences can also depend on the sender's preferences as well as the evidence, i.e. $U^R : T \times A \to \mathbb{R}$.

In addition to disclosing evidence, the sender can make a cheap talk declaration

[32] This formulation is also present in Ben-Porath et al. (2017). In that paper, the sender can prefer either higher or lower actions, and the receiver puts some type dependent weight on the sender's preferences. By arbitrarily adjusting this type dependent weight, one can create any mapping from sender types to best response actions for the receiver.

of his preference type. The set of available messages to each type t is $\{s : t \succeq'_d s\} \times \{H, L\}$. That is messaging is given by a preorder $\tilde{\succeq}_d$ defined by $(t,\theta)\tilde{\succeq}_d(t',\theta')\ \forall\theta,\theta' \in \{H,L\} \iff t \succeq'_d t'$.[33] Call this communication game $\tilde{C}$, and let $\pi^{\tilde{C}} : T \to \mathbb{R}$ be the corresponding ROE allocation of actions to types. Note that $\tilde{C}$ does not fit into the framework of Section 2.

7.1.1. An Equivalent Disclosure Game

Consider a related disclosure game, in which the sender has known preferences towards higher actions. The type space, $T \equiv T' \times \{H, L\}$, and type distribution $h \in \Delta T$ remain unchanged from $\tilde{C}$. Define a new disclosure order $\succeq_d$ from $\succeq'_d$ as follows:

$$
\begin{aligned}
(t, H) \succeq_d (t', H) &\iff t \succeq'_d t', \\
(t, L) \succeq_d (t', L) &\iff t' \succeq'_d t, \\
(t, H) \succeq_d (t', L) &\iff \exists s : t \succeq'_d s, \text{ and } t' \succeq'_d s.
\end{aligned}
$$

The disclosure order $\succeq_d$ (i) maintains $\succeq'_d$ when comparing two H types, (ii) reverses $\succeq'_d$ when comparing two L types, and (iii) ranks an H type above an L when both types can mimic some common evidence type $s \in T'$. Call the associated game $\tilde{D}$ with $\pi_{h_p}(t|U^R)$ as the ROE allocation of actions to sender types as described in the main text. I illustrate the construction of $(T, \succeq_d)$ in Figure 8 when $(T', \succeq'_d)$ is the Dye model example in Figure 1.

Proposition 6. *The receiver optimal equilibrium allocation in $\tilde{D}$ is the same as that in $\tilde{C}$, i.e. $\pi_{h_p}(t|U^R) = \pi^{\tilde{C}}(t),\ \forall t \in T$.*

Since $\tilde{D}$ fits into the framework of Section 2, Theorem 1 applies to $\pi^{\tilde{C}}(t)$. In $\tilde{D}$, increasing p constitutes a more evidence change in the distribution over $(T, \succeq_d)$.

Corollary 4. *Let $1 > p > p' > 0$ with $h_p, h_{p'} \in \Delta T$ as the corresponding prior distributions. $h_p \geq_{ME} h_{p'}$ on $(T, \succeq_d)$ so $h_p \geq_{MS} h_{p'}$.*

[33] Unlike in the original model, the inclusion of cheap talk messages can alter the set of equilibria. With both H and L senders, two on-path cheap talk messages can lead to different actions: H senders induce the high action and L senders induce the low action. For this reason, I include the possibility of cheap talk about preferences. Alternatively, the support of f_H and f_L may differ which allows the possibility that the sender can prove their taste type.

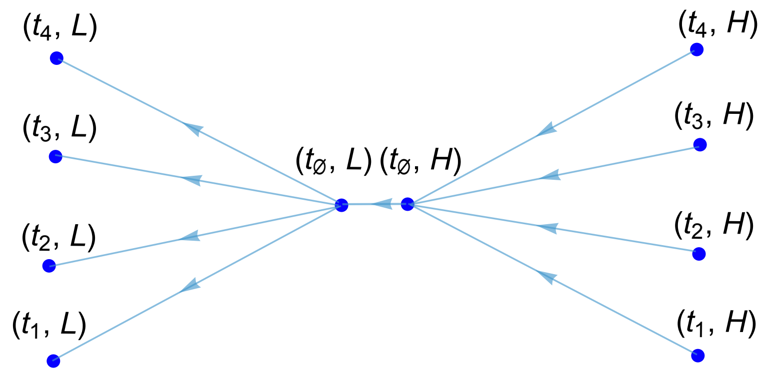


Figure 8: $(T, \succeq_d)$ in $\tilde{D}$ when $(T', \succeq'_d)$ is Dye evidence.

Note that more skepticism – a decrease in ROE actions – no longer corresponds to a decrease in utility. Indeed, the L type senders are better off facing a receiver with a more skeptical prior. Thus L types benefit from the presence of H types and vice versa. However, this comparative static would be irrelevant if the sender always revealed his preference. The construction of $\tilde{D}$ reveals that this is not always the case.

Proposition 7. *Let $(T', \succeq'_d)$ have a lower bound $\underline{s}$.*

1. *There is at most one on path equilibrium action at which H and L types pool and this pooled set includes $\underline{s}$.*
2. *If $f^H(t) = f^L(t)$, and $U^R(a, (t, H)) = U^R(a, (t, L))$, $\forall t \in T'$ then there is exactly one on path ROE action at which H and L types pool.*

This result says that if the value and distribution of evidence is independent of the sender's preferences then there is exactly one pooled set in which the sender does not reveal his preference. Outside of this pooled set, H and L type senders completely separate. Figure 8 elucidates this result: clearly at most one part of an interval partition of $(T, \succeq_d)$ can intersect "both sides" of the directed graph. If instead there were no part intersecting both sides, then all H types would obtain higher actions than all L types. However in part 2. of Proposition 7, the receiver's preferences and distribution of sender types is assumed to be independent of the sender's preference type. With these independence conditions, such separation cannot be optimal for the receiver.

The existence of $\underline{s}$ is a relatively innocuous assumption: I do not assume it has positive probability; the assumption only ensures its availability as a message. The presence of $\underline{s}$ can be justified by noting that the sender can always choose to present nothing. Note that unlike in the original model, the presence of zero probability types *does* affect the ROE, as it can change the disclosure order $\succeq_d$ in $\tilde{D}$.[34]

7.2. Unbiased vs. Honest Senders

Another reason the sender may not always prefer higher actions is that he may have a reason to act in the interest of the receiver. For example, criminal investigations are (hopefully) carried out by "good cops" whose goal is to find out the truth rather than to always convict the suspect. One can interpret this "goodness" as originating from either a compulsion to be honest or from preferences that are aligned with the court, and ask whether these two explanations lead to different outcomes. Kim and Pogach (2014) show that the answer to this question is yes in a standard cheap talk setting a la Crawford and Sobel (1982): depending on the specification, the receiver can prefer either honest or unbiased senders. I show that in the disclosure setting, unbiased and honest senders lead to the same ROE outcomes.

Let $\tilde{H}$ and $\tilde{UB}$ be two communication games defined as follows. In both games there is probability p that the sender prefers higher actions (type S for strategic). These senders have evidence from T' distributed according to $f^S \in \Delta T'$, and can send messages in T' according to $\succeq_d$. In both $\tilde{H}$ and $\tilde{UB}$, there is probability $1-p$ of non-strategic types (NS type), who have evidence from T' distributed according to $f^{NS} \in \Delta T'$. The total types space is denoted $T \equiv T' \times \{S, NS\}$ with unconditional distribution $h \in \Delta T$.

In $\tilde{H}$, the non-strategic sender is honest (H type) in that he can only declare truthfully. In $\tilde{UB}$, the non-strategic sender can disclose according to $\succeq_d$, but is unbiased (UB type) in that he has the same utility as the receiver. In both games

[34] This point relates to Seidmann and Winter (1997) who study a vagueness model in which the sender's direction of bias is also unknown. Even though the sender is "informed" with probability 1, the ability to be vague, i.e. mimic zero probability types, destroys the truthful revelation equilibrium.

the receiver's utility is dependent only on the evidence: $U^R : T' \times A \to \mathbb{R}$.[35]

$\tilde{H}$ fits the basic framework (it is the example from Subsection 2.1), while $\tilde{UB}$ does not. Thus the ROE outcomes in $\tilde{H}$ are given by Theorem 2. The following definition further refines this characterization in honest type games. For any $R \subset T'$, define

$$\tilde{V}(R) \equiv \max_{R' \subset R} V_h((R \times \{S\}) \cup (R' \times \{NS\})). \tag{8}$$

This is the receiver's best response to the biased senders in R and the honest or unbiased senders in some subset $R' \subset R$. R' is the set of types in R who have higher value than $\tilde{V}(R)$, i.e. $R' \equiv \{s \in R : v(s) \geq \tilde{V}(R)\}$. Mechanically, $\tilde{V}(R) \geq V_h(R),\ \forall R \subset T'$.

Proposition 8. *Let the receiver optimal equilibrium allocation in $\tilde{H}$ and $\tilde{UB}$ be given by π^H, π^{UB} respectively. The receiver optimal equilibrium allocations are the same, i.e. $\pi^H = \pi^{UB} \equiv \pi^*$. Moreover $\forall t \in T'$*

$$\pi^*((t, S)) = \min_{\{S_a \subset T' :\, t \in S_a\}} \max_{\{S_b \subset T' :\, t \in S_b\}} \tilde{V}(W(S_a) \cap B(S_b)),$$
$$\pi^*((t, NS)) = \min\{\pi^*((t, S)), v(t)\}. \tag{9}$$

The ROE actions for strategic types are the same as in a standard disclosure game without the non-strategic senders, but where the receiver has more favorable beliefs about the sender: his best response to all subsets shifts up from V_h to $\tilde{V}$. On the other hand the receiver obtains his bliss point for any nonstrategic sender with value less than the ROE action of his strategic counterpart.

Corollary 5. *$\pi^*(t)$ is increasing in p $\forall t \in T$.*

The result says that the receiver is less skeptical when the sender has a higher probability of being honest or unbiased. This is a direct corollary of Theorem 1 as increasing the probability of honest types induces a distribution with less evidence.

[35] In the case where U^R is quadratic loss and the sender's utility in linear in the action, one can show that the game where the sender is honest with probability p has equivalent equilibria to one in which the receiver is naive with probability p. The receiver is naive in the sense of Ottaviani and Squintani (2006), i.e. takes action $v(t)$ when type t is declared. For any strategic sender strategy, both games induce the same expected receiver best response, and thereby have the same mapping from reported types to actions.

7.3. Skepticism with Restricted Receiver Preferences

A more skeptical distribution induces lower ROE actions for all types, regardless of the receiver's preferences. The latter condition may be overly demanding in scenarios in which it is known that the receiver will value certain types higher than others. For example, any investor would assign a higher value to an entrepreneur whose customer reviews are positive rather than negative. This section generalizes Theorem 1 to characterize a version of more skepticism in which the inequality in (1) need only hold for certain receiver utilities.

Consider an arbitrary partial order $\succeq_v$ on T. For readability, I assume that all distributions have full support. Define the restricted set of receiver utilities $\Upsilon_{\succeq_v} \equiv \{U^R \in \Upsilon : \ t \succeq_v t' \implies v(t) \geq v(t')\}$. $\Upsilon_{\succeq_v}$ is the set of receivers who take a higher action for types ranked higher according to $\succeq_v$.

Definition 6. Let $f, g \in \Delta T$. f is more skeptical than g for receivers who agree on $\succeq_v$ if,

$$\pi_f(t|U^R) \leq \pi_g(t|U^R), \ \forall t \in T, \ \forall U^R \in \Upsilon_{\succeq_v}.$$

Define $\succeq_{d,v}$ over T as the transitive closure of the relation given by $t \succeq_d t'$ and $t \not\succeq_v t'$.[36] The interpretation is that $t \succeq_{d,v} t'$, if t not only has the ability to mimic t', but also the incentive, for some receiver preferences in $\Upsilon_{\succeq_v}$. The next result shows that the more evidence relation restricted to comparisons in $\succeq_{d,v}$ characterizes when receivers who agree on $\succeq_v$ are more skeptical.

Theorem 3. *f is more skeptical than g for receivers who agree on $\succeq_v$ if and only if $f \geq_{ME} g$ with respect to $(T, \succeq_{d,v})$.*

7.3.1. More Skeptical Information Structures

Consider the situation where there is some underlying payoff relevant unknown state, and an analyst seeks to compare the outcomes of two senders whose evidence distributions induce different information structures. In these contexts, applications usually fix the "good states" and "bad states", i.e. which states the receiver values higher than others. With some reframing, one can use Theorem 3 to deal with this restriction on the set of admissible receiver preferences.

Let the state space be X with prior distribution $p \in \Delta X$. The receiver has utility given by $U^R : A \times X \to \mathbb{R}$. The sender obtains evidence in T' drawn according to

[36] The transitive closure of a binary relation is its coarsest transitive refinement.

the *information structure*, $pr(t|x) \equiv q_x(t)$, which can be disclosed according to $\succeq'_d$. Since the sender cannot credibly reveal the state, it is without loss to assume he knows it, i.e. $T \equiv T' \times X$. Then messaging is given by the preorder, $\succeq_d$, defined by $(t,x) \succeq_d (t',x')$ if $t \succeq'_d t'$.

Recall that it is without loss to focus on the equivalence classes under $\succeq_d$, i.e. $(T', \succeq'_d)$ and define $U^R(a,t) \equiv \frac{1}{\sum_{x\in X} q_x(t)p(x)} \sum_{x\in X} U^R(a,x)q_x(t)p(x)$ as the induced receiver utility, and $h(t) \equiv \sum_{x\in X} q_x(t)p(x)$ as the induced prior distribution over evidence. While this framework will obtain the correct ROE, it is not as apparently useful for thinking about comparative statics in the information structure. Changes in the information structure q change *both* the induced distribution over evidence h and the induced receiver utility $U^R(a,t)$. This means we cannot directly compare two information structures using Theorem 1.

However, this can be remedied by considering the pre-ordered type space $(T, \succeq_d)$. In this framework, Theorem 1 tells us that one information structure is more skeptical than another if it has more evidence with respect to $(T, \succeq_d)$. Since, for every pair of states x, x' and evidence type t, $(t,x) \sim (t,x')$, the more evidence restriction implies that the relative probability of states given each evidence signal t must remain constant. That is, the more evidence relation in this context imposes that the two information structures induce the same posteriors about the state for every evidence signal. One reason for this strong restriction is that the definition of more skepticism allows complete flexibility in the receiver's preferences, which as mentioned above, may be ill-fit for applications. If instead, the receiver's preference over states is fixed, we can refine the above statement using Theorem 3. Label $X = (x_1, ..., x_m)$ and assume that the receiver prefers higher actions in higher states, i.e. $v(x_1) < ... < v(x_m)$. Define $\succeq_v$ as $(t, x_i) \succeq_v (t', x_j)$ if $i \geq j$.

Corollary 6. *Let $q^1, q^2 : X \to \Delta T'$ be two information structures over evidence signals with associated evidence distributions $h_i(t) \equiv \sum_{x\in X} q^i_x(t)p(x)$. q^1 induces more skepticism than q^2 for receiver's who agree on $\succeq_v$ if*

$$\frac{q^1_{x_i}(t)}{q^1_{x_j}(t)} \leq \frac{q^2_{x_i}(t)}{q^2_{x_j}(t)} \quad \forall i > j,\ \forall t \in T',\ \textit{and} \tag{10}$$

$$\frac{h_1(t)}{h_1(t')} \geq \frac{h_2(t)}{h_2(t')} \quad \forall t, t' \in T' :\ t \succeq'_d t'. \tag{11}$$

The result says that if we fix the receiver's preferences over states then any change in the information structure that (i) decreases the posterior in a likelihood ratio sense for each evidence signal and (ii) increases the relative probability of evidence signals that are more dominant, induces lower ROE equilibrium actions for every evidence signal. Notice that the evidence signals that are more dominant according to $\succeq_d'$ need not be the ones with less favorable posteriors.

Unlike in Theorem 1, I do not provide a characterization of the changes in the information structure that induce lower equilibrium actions for all $U^R(a, x)$. The reason is that the ROE varies less with $U^R(a, x)$ than with $U^R(a, t)$. If $U^R(a, t)$ can be chosen arbitrarily, then, roughly, so can the ROE equilibrium pooling behavior. This is what necessitates the more evidence comparison in order for two prior beliefs to be comparable according to the more skepticism order. Conversely, consider the case in which $X = \{x_1, x_2\}$ is binary. In this case, there are only two possibilities for the ROE when varying $U^R(a, x)$ depending on how $v(x_1)$ compares to $v(x_2)$.

8. Conclusion

This paper has two main contributions: (i) it characterizes the receiver optimal equilibrium in a large class of verifiable disclosure games and (ii) it shows that distributions which induce greater skepticism in the receiver are characterized by the more evidence relation. This comparative statics result unifies existing conculsions in the literature that are used in a variety of signaling applications. In addition, the more skepticism characterization can open up unanswered signaling questions that necessitate studying a multidimensional evidence structure as exemplified in Section 6.

References

Acharya, V. V., DeMarzo, P., and Kremer, I. (2011). Endogenous information flows and the clustering of announcements. *American Economic Review*, 101(7):2955–79.

Ben-Porath, E., Dekel, E., and Lipman, B. (2017). Mechanisms with evidence: Commitment and robustness. *Working Paper*.

Bertomeu, J. and Cianciaruso, D. (2016). Verifiable disclosure. *Available at SSRN*.

Bhattacharya, S. and Mukherjee, A. (2013). Strategic information revelation when experts compete to influence. *The RAND Journal of Economics*, 44(3):522–544.

Bull, J. and Watson, J. (2004). Evidence disclosure and verifiability. *Journal of Economic Theory*, 118(1):1 – 31.

Crawford, V. and Sobel, J. (1982). Strategic information transmission. *Econometrica*, 50(6):1431–1451.

Dranove, D. and Jin, G. Z. (2010). Quality disclosure and certification: Theory and practice. *Journal of Economic Literature*, 48(4):935–963.

Dye, R. A. (1985). Disclosure of nonproprietary information. *Journal of Accounting Research*, 23(1):123–145.

Dziuda, W. (2011). Strategic argumentation. *Journal of Economic Theory*, 146(4):1362 – 1397.

Einhorn, E. (2007). Voluntary disclosure under uncertainty about the reporting objective. *Journal of Accounting and Economics*, 43(2):245 – 274.

Glazer, J. and Rubinstein, A. (2004). On optimal rules of persuasion. *Econometrica*, 72(6):1715–1736.

Green, J. R. and Laffont, J.-J. (1986). Partially verifiable information and mechanism design. *Review of Economic Studies*, 53(3):447–456.

Grossman, S. (1981). The informational role of warranties and private disclosure about product quality. *Journal of Law and Economics*, 24(3):461–83.

Grossman, S. J. and Hart, O. D. (1980). Disclosure laws and takeover bids. *The Journal of Finance*, 35(2):323–334.

Grubb, M. D. (2011). Developing a reputation for reticence. *Journal of Economics and Management Strategy*, 20(1):225–268.

Guttman, I., Kremer, I., and Skrzypacz, A. (2014). Not only what but also when: A theory of dynamic voluntary disclosure. *American Economic Review*, 104(8):2400–2420.

Hagenbach, J., Koessler, F., and Perez-Richet, E. (2014). Certifiable pre-play communication: Full disclosure. *Econometrica*, 82(3):1093–1131.

Hart, S., Kremer, I., and Perry, M. (2017). Evidence games: Truth and commitment. *American Economic Review*, 107(3):690–713.

Jung, W.-O. and Kwon, Y. K. (1988). Disclosure when the market is unsure of information endowment of managers. *Journal of Accounting Research*, 26(1):146–153.

Kim, K. and Pogach, J. (2014). Honesty vs. advocacy. *Journal of Economic Behavior Organization*, 105(Supplement C):51 – 74.

Mathis, J. (2008). Full revelation of information in sender–receiver games of persuasion. *Journal of Economic Theory*, 143(1):571 – 584.

Milgrom, P. (2008). What the seller won't tell you: Persuasion and disclosure in markets. *Journal of Economic Perspectives*, 22(2):115–131.

Milgrom, P. R. (1981). Good news and bad news: Representation theorems and applications. *Bell Journal of Economics*, 12(2):380–391.

Ottaviani, M. and Squintani, F. (2006). Naive audience and communication bias. *International Journal of Game Theory*, 35:129–150.

Seidmann, D. J. and Winter, E. (1997). Strategic information transmission with verifiable messages. *Econometrica*, 65(1):163–169.

Shelton, D., Kim, Y., and Barak, G. (2009). Examining the "csi-effect" in the cases of circumstantial evidence and eyewitness testimony: Multivariate and path analyses. *Journal of Criminal Justice*, 37(5):452–460.

Sher, I. (2011). Credibility and determinism in a game of persuasion. *Games and Economic Behavior*, 71(2):409 – 419.

Sher, I. (2014). Persuasion and dynamic communication. *Theoretical Economics*, 9(1):99–136.

Shin, H. S. (2003). Disclosures and asset returns. *Econometrica*, 71(1):105–133.

Topkis, D. M. (1976). The structure of sublattices of the product of n lattices. *Pacific J. Math.*, 65(2):525–532.

Verrecchia, R. E. (1983). Discretionary disclosure. *Journal of Accounting and Economics*, 5:179 – 194.

A. Preliminaries

First I state some results that will be useful in the arguments to follow. Their proofs are deferred to the online appendix.

A.1. In-betweenness

Lemma 2. *For any distribution over types $q \in \Delta T$, Consider two distributions $q_1, q_2 \in \Delta T$ such that $V_{q_1}(T) < V_{q_2}(T)$. For any $\lambda \in (0,1)$, $V_{q_1}(T) < V_{\lambda q_1 + (1-\lambda) q_2}(T) < V_{q_2}(T)$. In particular for two disjoint subsets $S', S'' \subset T$, and $h \in \Delta T$, $V_h(S') \leq V_h(S'')$ implies $V_h(S') \leq V_h(S' \cup S'') \leq V_h(S'')$.*

Proof. Let $a_i \equiv V_{q_i}(T)$ for $i = 1, 2$. Note that for any distribution $q \in \Delta T$ and $\tilde{a} \in \mathbb{R}$, because U^R is strictly concave, $V_q(T) \leq (\geq)\tilde{a} \iff \sum_{t \in T} U_a^R(\tilde{a}, t) q(t) \leq (\geq) 0$,

$\sum_{t \in T} U_a^R(a_2, t) q_1(t) < 0$ and $\sum_t U_a^R(a_1, t) q_2(t) > 0$. This implies that

$$\begin{aligned}
&\text{sign}\left[a^*(\lambda q_1 + (1-\lambda) q_2) - a_i\right] \\
=&\text{sign}\left[\sum_{t \in T} U_a^R(a_i, t)(\lambda q_1(t) + (1-\lambda) q_2(t))\right] \\
=&\text{sign}\left[\lambda \sum_{t \in T} U_a^R(a_i, t) q_1(t) + (1-\lambda) \sum_{t \in T} U_a^R(a_i, t) q_2(t)\right] \\
=&\text{sign}\left[\sum_{t \in T} U_a^R(a_i, t) q_{-i}(t)\right].
\end{aligned}$$

Where the last equality follows from optimality of a_i with respect to q_i. By the fact above $\sum_{t \in T} U_a^R(a_2, t) q_1(t) < 0$ and $\sum_{t \in T} U_a^R(a_1, t) q_2(t) > 0$. Thus if $i = 1$ ($i = 2$) then this expression is strictly positive (negative) thereby proving the result. *Q.E.D.*

A.2. Poolable Sets

Let $(P_1, ..., P_m)$ be an (not necessarily receiver optimal) equilibrium partition of T under distribution h. For each P_i, there must exist a sender strategy that induces the receiver to take the same action following each on-path declaration within P_i. I characterize when such a strategy exists.

Definition 7. Let $(S, \succeq_d)$ be a partially ordered set. $(S, \succeq_d)$ is *poolable* with respect to $h \in \Delta S$ if there exists a sender strategy $\sigma : S \to \Delta S$ such that $\text{Supp}(\sigma_s) \subset W_{\succeq_d}(s)\ \forall s \in S$ and receiver best responses on path $a^\sigma : \cup_{s \in S} \text{Supp}(\sigma_s) \to A$ such that $a^\sigma(s) = V_h(S)\ \forall s \in \cup_{s \in S} \text{Supp}(\sigma_s)$.

Define the following useful notation for a partially ordered set $(S, \succeq_d)$:

$$\begin{aligned}
&(i)\ \underline{W}(S) \equiv \{t \in S : \ \forall s \in S\ t \not\succ_d s\}, \\
&(ii)\ V^+(S) \equiv \{s \in S : \ v(s) \geq V_h(S)\},\ and \\
&(iii)\ \forall W' \subset \underline{W}(S)\ E(W') \equiv B(W') \setminus B(W(S) \setminus W'),\ and \\
&(iv)\ \forall W' \subset \underline{W}(S)\ Q(W') \equiv E(W') \cup (B(W') \cap V^+(S)).
\end{aligned}$$

Again when the order is ambiguous, I subscript each of these notations with the particular order. The set of non-dominant types in S are $\underline{W}(S)$. In searching for pooling strategies it is without loss to focus on those whose support is contained in $\underline{W}(S)$. $V^+(S)$ are the set of types who have value greater than the average value of S. $E(W')$ are the set of types that cannot declare non-dominant types outside of W'. In any pooling strategy these types must support their strategy on W'. Finally $Q(W')$ is the combination of $E(W')$ and types that can make declarations in W' and have higher value than S.

Lemma 3. *a partially ordered set.$(S, \succeq_d)$ is poolable with respect to $h \in \Delta S$ if and only if*

$$\forall W' \subset \underline{W}(S) \quad V_h(Q(W')) \geq V_h(S). \tag{12}$$

Proof. " $\Longrightarrow$ "

Take an arbitrary $W' \subset \underline{W}(S)$. Since σ is pooling the best response must be $a^\sigma(w) = V_h(S)\ \forall w \in \text{Supp}(\sigma) \subset \underline{W}(S)$. Let

$$q(t) \equiv \frac{\sum_{s \in \underline{W}(S)} \sigma_t(s) h(t)}{\sum_{t \in T} \sum_{s \in \underline{W}(S)} \sigma_t(s) h(t)},$$

and note that by Lemma 2 and the fact that σ is pooling, $V_q(S) = V_h(S)$. Also note that $q(t) = h(t)\ \forall t \in E(W')$. Moreover, the induced distribution of types given $Q(W')$ adds probability of types with value greater than $V_h(S)$ and decreases the probability of types with value less than $V_h(S)$, by Lemma 2 it must be that $V_h(S) \leq V_h(Q(W'))$.

" $\Longleftarrow$ "

Let $\mathcal{H} \subset \Delta S \equiv \{g \in \Delta S : \ \underline{W}(\text{Supp}(g)) \subset \underline{W}(S)\}$. Clearly $h \in \mathcal{H}$. I show that there exists a pooling strategy on $(S, \succeq_d)$with respect to every $g \in \mathcal{H}$ by induction on $|\underline{W}(\text{Supp}(g))|$. For the base case of $\underline{W}(\text{Supp}(g)) = \{w\}$, the strategy $\sigma_t(w) = 1\ \forall t \in S$ is a pooling strategy.

Now let there exist pooling strategies for all $g \in \mathcal{H}$ that satisfy (12) on $\underline{W}(\text{Supp}(g))$, such that $|\underline{W}(\text{Supp}(g))| = N$ and consider a distribution $g' \in \Delta S$ with $|\underline{W}(\text{Supp}(g'))| = N + 1$ that satisfies (12) on $\underline{W}(\text{Supp}(g'))$. Consider arbitrary $w \in \underline{W}(\text{Supp}(g'))$. First consider the case in which $V_{g'}(E(\{w\})) \leq V_{g'}(S)$. For $\lambda \in [0, 1]$ define C_λ and distribution $f_\lambda \in \Delta S$ as follows

$$C_\lambda \equiv \sum_{s \in S} \left(\lambda \mathbb{1}_{s \in Q(\{w\})} + (1 - \lambda) \mathbb{1}_{s \in E(\{w\})}\right) g'(s),$$

$$f_\lambda(s) \equiv \left(\lambda \mathbb{1}_{s \in Q(\{w\})} + (1 - \lambda) \mathbb{1}_{s \in E(\{w\})}\right) g'(s)/C_\lambda\ \forall s \in S.$$

First, note that because $E(\{w\}) \subset Q(\{w\})$ $f_\lambda(s) = g'(s)/C_\lambda$ $\forall s \in E(\{w\})$, i.e. f_λ includes all probability mass of types that must declare w. Second, note that $f_1 = g'_{|Q(\{w\})}$ so $V_{f_1}(S) = V_{g'}(Q(\{w\}))$ which means that $V_{f_1}(S) \geq V_{g'}(S)$ by (12). Third, note that $f_0 = g'_{|E(\{w\})}$ so $V_{f_0}(S) = V_{g'}(E(\{w\}))$ which means that $V_{f_0}(S) \leq V_{g'}(S)$ by assumption. Since the receiver's best response is continuous in λ, $\exists \lambda \in [0,1]$ such that $V_{f_\lambda}(S) = V_{g'}(S)$.

Suppose now that $V_{g'}(E(\{w\})) \geq V_{g'}(S)$. Notice that by definition $Q(W \setminus \{w\}) \cap E(\{w\}) = \emptyset$ and $V_{g'}(Q(W \setminus \{w\})) \geq V_{g'}(S)$ by (12). Let $R \equiv Q(W \setminus \{w\})^c$. This means that $E(\{w\}) \subset R$ and $V_{g'}(R) \leq V_{g'}(S)$ by Lemma 2. Now for $\lambda \in [0,1]$ redefine

$$C_\lambda \equiv \sum_{s \in S} \left(\lambda \mathbb{1}_{s \in R} + (1-\lambda)\mathbb{1}_{s \in E(\{w\})}\right) g'(s),$$

$$f_\lambda(s) \equiv \left(\lambda \mathbb{1}_{s \in R} + (1-\lambda)\mathbb{1}_{s \in E(\{w\})}\right) g'(s)/C_\lambda \ \forall s \in S.$$

By symmetric logic to the above paragraph $f_\lambda(s) = g'(s)/C_\lambda$ $\forall s \in E(\{w\})$ and $V_{f_1}(S) \leq V_{g'}(S) \leq V_{f_0}(S)$. Since the receiver's best response is continuous in λ, $\exists \lambda \in [0,1]$ such that $V_{f_\lambda}(S) = V_{g'}(S)$.

Now consider the distribution, $g'' \in \Delta S$ defined by $g''(s) \equiv \dfrac{g'(s) - f_\lambda(s)C_\lambda}{1 - C_\lambda}$. Since g' is a convex combination of g'' and f_λ and $V_{f_\lambda}(S) = V_{g'}(S)$, by Lemma 2 $V_{g''}(S) = V_{g'}(S)$. By definition of f_λ, $g''(s) = 0$ $\forall s \in E(\{w\})$, so $\underline{W}(\mathrm{Supp}(g'')) \subset \underline{W}(S) \setminus \{w\}$. Thus $|\underline{W}(\mathrm{Supp}(g''))| = N$.

Now I verify that S with distribution g'' satisfies (12). Consider arbitrary $W' \subset \underline{W}(\mathrm{Supp}(g''))$. By construction $\mathrm{Supp}(f_\lambda) \subset Q(\{w\}) \subset Q(W' \cup \{w\})$. Thus $V_{f_\lambda}(Q(W' \cup \{w\})) = V_{f_\lambda}(S) = V_{g'}(S)$. By (12) on S with respect to g', $V_{g'}(Q(W' \cup \{w\})) \geq V_{g'}(S)$. Finally, since g' is a convex combination of g'' and f_λ, by Lemma 2 $V_{g''}(Q(W' \cup \{w\})) = V_{g''}(Q(W')) \geq V_{g'}(S) = V_{g''}(S)$ and (12) is satisfied on S with respect to g''.

By the induction hypothesis, $\exists \sigma : \mathrm{Supp}(g'') \to \Delta \underline{W}(\mathrm{Supp}(g''))$ that is pooling with respect to g''. Now $\forall w' \in \underline{W}(S)$ define

$$\tilde{\sigma}_{t'}(w') \equiv \begin{cases} (1 - C_\lambda f_\lambda(t'))\sigma_{t'}(w') & \text{if } w' \in \underline{W}(\mathrm{Supp}(g'')) \\ C_\lambda f_\lambda(t') & \text{if } w' = w \end{cases}.$$

The best responses to $\tilde{\sigma}$ are the best responses to σ on $W(\mathrm{Supp}(g''))$ and the best response to f_λ on w. Thus $\tilde{\sigma}$ is a pooling strategy for g' on S. *Q.E.D.*

Lemma 4. *If V_h is downward biased on $(S, \succeq_d)$ then $(S, \succeq_d)$ is poolable with respect to h.*

Proof. Take an arbitrary subset of non-dominant types in $(S, \succeq_d)$, $W' \subset \underline{W}(S)$. Because $E(W')$ is a lower contour subset of S and V_h is downward biased on S, $V_h(E(W')) \geq V_h(S)$. By Lemma 2 $V_h(Q(W')) \geq V_h(S)$. The result follows from Lemma 3. *Q.E.D.*

B. Proofs of Main Results

B.1. Proof of Proposition 1

Proof. I use Algorithm 1 to construct an interval partition $P^* = (P_1^*, ..., P_{m^*}^*)$ such that V_h is downward biased on P_i^* $\forall i = 1, ..., m$. I will show that such a partition is the ROE partition. The existence of pooling strategies on each P_i^* is provided by Lemma 4.[37,38]

Suppose that $\pi : T \to \mathbb{R}$ is some alternative allocation such that $t \succeq_d t' \implies \pi(t) \geq \pi(t')$. Let $P = (P_1, ..., P_m)$ represent the ordered equivalence classes induced by π, i.e. $P_i = \pi^{-1}(\pi_i)$ where π_i is the $i'th$ highest value in the range of π. Note that P is an interval partition. Otherwise $t' \succeq_d t''$ where $t' \in P_i$, $t'' \in P_j$, and $j > i$. But then t' can deviate to the strategy of t'' and obtain a strictly higher action. I prove the more general result.

Claim 1. *The receiver's utility is higher under P^* than under π.*

Proof of Claim: I will show that the receiver's utility is higher on each part P_i^*. Let $Q_j \equiv P_l \cap P_i^*$ and $a_j \equiv \pi_l$ where l is the j'th highest index such that $P_l \cap P_i^* \neq \emptyset$. Note that $P_i^* = \cup_{j=1}^{\overline{k}} Q_j$ for some $\overline{k} \geq 1$. Take $\hat{k}$ such that $a_1 < ... < a_{\hat{k}} \leq V_h(P_i^*) \leq a_{\hat{k}+1} < ... < a_{\overline{k}}$. Note that because P is an interval partition, $\cup_{j=1}^{k} Q_j \equiv \underline{Q}_k$ is a lower contour subset of P_i^* for every k. This means that $V_h(\underline{Q}_k) \geq V_h(P_i^*)$ $\forall k$ because V_h is downward biased on each P_i^*. By strict concavity of U^R, moving the action for any

[37] For any off path $t' \in P_i^*$ in such a pooling strategy one can set $a(t) = V_h(B(t) \cap S) \leq V_h(P_i^*)$ where the inequality follows for the fact that V_h is downward biased on P_i^*.

[38] The existence of pooling strategies, given the proceeding argument, could be alternatively guaranteed by the equivalence between the commitment allocation and the ROE allocation. Lemma 3 characterizes the existence of a pooling strategy generally and could be of independent interest.

set closer to the bliss point for that set increases the receiver's utility. Thus $\forall k \leq \hat{k}$,

$$\sum_{t\in \underline{Q}_k} U^R(a_k,t)h(t) \leq \sum_{t\in \underline{Q}_k} U^R(a_{k+1},t)h(t) \leq \sum_{t\in \underline{Q}_k} U^R(V_h(P_i^*),t)h(t). \tag{13}$$

Using the first inequality in (13) gives that $\forall k \leq \hat{k}$,

$$\sum_{t\in \underline{Q}_k} U^R(a_k,t)h(t) + \sum_{j=k+1}^{\hat{k}} \sum_{t\in Q_j} U^R(a_j,t)h(t) \leq \sum_{t\in \underline{Q}_{k+1}} U^R(a_{k+1},t)h(t) + \sum_{j=k+2}^{\hat{k}} \sum_{t\in Q_j} U^R(a_j,t)h(t)$$

A sequence of these inequalities as k ranges from 1 to $\hat{k}-1$ gives that

$$\sum_{j=1}^{\hat{k}} \sum_{t\in Q_j} U^R(a_j,t)h(t) \leq \sum_{t\in \underline{Q}_{\hat{k}}} U^R(a_{\hat{k}},t)h(t) \leq \sum_{t\in \underline{Q}_{\hat{k}}} U^R(V_h(P_i^*),t)h(t).$$

where the second inequality comes from the second inequality in (13). This shows that the receiver does better on $Q_1 \cup ... \cup Q_{\hat{k}}$ under P^* than under P. A symmetric argument shows that the receiver also does better on $\cup_{j=\hat{k}+1}^{\overline{k}} Q_j$. In addition, if P is a different partition than P^* then there will exist i and j such that $P_i^* \cap P_j \neq \emptyset$ and $V_h(P_j) \neq V_h(P_i^*)$ making at least one of the above inequalities strict. *Q.E.D.*

B.2. Proof of Lemma 1

Proof. Let $S^* \in \arg\min_{\tilde{S}\subset S} V_h(W(\tilde{S}) \cap S)$ with value $\overline{V}$, let $\overline{W} \equiv W(S^*) \cap S$. I first prove that, $V_h(W(\tilde{S}) \cap \overline{W}) \geq V_h(\overline{W}),\ \forall \tilde{S} \subset \overline{W}$. Suppose not, and take $\overline{W}' \equiv W(\tilde{S})\cap\overline{W}$ such that $V_h(\overline{W}') < V_h(\overline{W})$. Note that $W(\overline{W}')\cap S = \overline{W}'$, which contradicts the minimality of $\overline{W}$ in the above problem. Thus each minimizer of the above problem is downward biased.

Now take $J \subset \arg\min_{\tilde{S}\subset S} V_h(W(\tilde{S}) \cap S)$ with $J \equiv (S_1, ..., S_k)$, $\overline{W}_i \equiv W(S_i) \cap S$, and $\overline{W} \equiv \cup_{i=1}^k \overline{W}_i$. Note that because each $\overline{W}_i$ is downward biased, for each i, $V_h(\overline{W}_i \setminus \cup_{j=1}^{i-1}\overline{W}_j) \leq V_h(\overline{W}_i) = \overline{V}$. Since $\overline{W}$ is the disjoint union of these sets, i.e. $\overline{W} = \cup_{i=1}^k (W_i \setminus \cup_{j=1}^{i-1}\overline{W}_j)$, Lemma 2 implies that $V_h(\overline{W}) \geq \overline{V}$. Thus $\overline{W} \in \arg\max_{\tilde{S}\subset S} V_h(W(\tilde{S}) \cap S)$, and so by the previous argument $\overline{W}$ is downward biased. The argument is symmetric for the $\arg\max$ case. *Q.E.D.*

B.3. Proof of Proposition 2

Proof. Algorithm 1 produces a partition of T into disjoint sets $(P_1, P_2, ..., P_m)$. I argue that this partition satisfies the requirements of Proposition 1, and thereby constitutes the ROE partition. Lemma 1 implies that each P_i is a downward biased set. One must only check that P is an interval partition. By construction $P_{i+1} \cup P_i$ is available in the minimization that yields P_i. Thus in order for P_i to be chosen at the i'th stage, $V_h(P_i) < V_h(P_{i+1})$ otherwise by Lemma 2 $V_h(P_i \cup P_{i+1}) \leq V_h(P_i)$. Thus the indexing on P already follows the convention that $V_h(P_i)$ is increasing in i. Now suppose that $t \succeq_d t'$ with $t \in P_i$, and $t' \in P_j$ such that $i < j$. Then $W(P_i) \cap (T \backslash (\cup_{k=1}^{i-1} P_i)) \neq P_i$ contradicting the fact that P_i was selected by the algorithm. The argument is symmetric for the $\arg\max$ case. *Q.E.D.*

B.4. Proof of Theorem 2

Proof. Take the ROE partition $(P_1, ..., P_m)$. For $t \in P_i$, $V_h(P_i) = \pi_h(t|U^R)$. Thus, I prove that the solution to the problem on the right hand side of (6) is $V_h(P_i)$. Define $S_a^* \equiv \cup_{k=1}^{i} P_k$ and $S_b^* \equiv \cup_{k=i}^{m} P_k$. I show that choosing $S_a = S_a^*$ bounds the value of (6) to be less than $V_h(P_i)$. If we instead consider

$$\max_{\{S_b: t \in S_b\}} \min_{\{S_a: t \in S_a\}} V_h(W(S_a) \cap B(S_b)). \tag{14}$$

The argument for why choosing $S_b = S_b^*$ bounds the max-min value in (14) to be greater than $V_h(P_i)$ is exactly symmetric. The conclusion follows from the max-min inequality.

Take any feasible S_b. $B(S_b) \cap W(S_a^*) = \cup_{k=1}^{i} (B(S_b) \cap P_k)$. By Proposition 1 and since $B(S_b) \cap P_k$ is an upper contour subset of each P_k, for $k \leq i$ (whenever non-empty) $V_h((B(S_b) \cap P_k) \leq V_h(P_k) \leq V_h(P_i)$. Thus, by Lemma 2, $V_h(\cup_{k=1}^{i} (B(S_b) \cap P_k)) \leq V_h(P_i)$. In words, by choosing $S_a = S_a^*$ the minimizer can achieve a value less than $V_h(P_i)$. Thus the value of (6) is less than $V_h(P_i)$. *Q.E.D.*

B.5. Proof of Proposition 3

Proof. " $\Longleftarrow$ "

Let $r : S \to \mathbb{R}$ be defined as $r(s) \equiv U_a^R(V_f(S), s)$ and define $E_h^r(S') \equiv \mathbb{E}[r(s')|s' \in S', s' \sim h]$. Notice that (i) $E_f^r(S) = 0$, and (ii) because U^R is strictly concave, $V_h(\tilde{S}) \geq V_f(S) \iff E_h^r(\tilde{S}) \geq 0, \ \forall \tilde{S} \subset S, \ \forall h \in \Delta S$. Because V_f is downward biased on

$(S, \succeq_d)$, this means that E^r_f is also downward biased on $(S, \succeq_d)$. Because $f \geq_{ME} g$, $(S, \geq_{f/g})$ (where we break ties according to $\succeq_d$ and arbitrarily otherwise) is a complete refinement of $(S, \succeq_d)$, and so E^r_f is also downward biased on $(S, \geq_{f/g})$. Now I will input $(S, \geq_{f/g})$ into the algorithm using the receiver's set valued best response as E^r instead of V.

Because the algorithm acts on a finite set and repeatedly returns strictly coarser partitions it must terminate at some stage T. At this point $Q^T = Q^{T+1}$ which means that $E^r_f(P^T_1) < ... < E^r_f(P^T_m)$. Thus Q^T must be trivial because E^r_f is downward biased on $(S, \geq_{f/g})$.

Consider the partition $Q^i = (Q^i_1, ..., Q^i_m)$ generated at stage $i > 1$. Each part Q^i_j is the union of an interval of parts from the previous partition Q^{i-1}. That is for each j there exists $\underline{k}(j) \leq \overline{k}(j)$ such that $Q^i_j = \cup_{l=\underline{k}(j)}^{\overline{k}(j)} Q^{i-1}_l$. Because Q^{i-1} is an interval partition of $(S, \geq_{f/g})$, and $E^r_f(Q^{i-1}_l)$ is decreasing for $\underline{k}(j) \leq l \leq \overline{k}(j)$ one can use Fact 1 on the set Q^i_j, to obtain that $\forall i, j$,

$$\frac{1}{G(Q^i_j)} \sum_{l=\underline{k}(j)}^{\overline{k}(j)} E^r_f(Q^{i-1}_l) G(Q^{i-1}_l) \geq \frac{1}{F(Q^i_j)} \sum_{l=\underline{k}(j)}^{\overline{k}(j)} E^r_f(Q^{i-1}_l) F(Q^{i-1}_l) = E^r_f(Q^i_l).$$

Using a string of these inequalities on each part of the interval partition at each stage of the algorithm we get,

$$E^r_g(S) = \sum_{Q^1_k \subset S} E^r_f(Q^1_k) G(Q^1_k) \geq \sum_{Q^T_k \subset S} E^r_f(Q^T_k) G(Q^T_k) = E^r_f(S).$$

Where the first equality follows from the fact that Q^1 is the complete partition on S and the second equality follows from the fact that $Q^T = (S)$ is the trivial partition on S.

" $\implies$ " Suppose V_f is not downward biased on $(S, \succeq_d)$. This means there exists a lower contour subset $L = W(L) \subset S$, such that $V_f(L) < V_f(S) \implies V_f(L) < V_f(S \setminus L)$. Define $g(s) = \frac{f(s)}{F(L)}$ if $s \in L$ and $g(s) = 0$ otherwise. $f \geq_{ME} g$ but $V_f(S) > V_g(S)$. *Q.E.D.*

B.6. Proof of Theorem 1

I actually prove the following stronger result:

Theorem 4. *Let $f, g \in \Delta T$, where $T = \text{Supp}(f) \cup \text{Supp}(g)$. $f \geq_{ME} g \implies$*

$$\pi_f(t|U^R) \leq \pi_g(t|U^R) \; \forall t \in \text{Supp}(f) \cap \text{Supp}(g), \; \forall U^R \in \Upsilon.$$

Moreover, if $f(t)g(t') < f(t')g(t)$ for some $t \succeq_d t'$ such that $t' \in \text{Supp}(f) \cap \text{Supp}(g)$ then $\exists U^R \in \Upsilon$ such that $\pi_f(t'|U^R) > \pi_g(t'|U^R)$.

Proof. " $\implies$ " Note that if $f \geq_{ME} g$ then $Z^g \equiv \{t \in T : g(t) = 0\}$ is an upper contour set of $(T, \succeq_d)$ and $Z^f \equiv \{t \in T : f(t) = 0\}$ is a lower contour set of $(T, \succeq_d)$, i.e. $B(Z_g) = Z_g$ and $W(Z_f) = Z_f$.Fix a receiver utility U^R. Let $P^g = (P_1^g, ..., P_l^g, Z^g)$ be the ROE partition under g and take arbitrary j and $t \in P_j^g$. Define $D^g \equiv \cup_{k=1}^{j} P_k^g$ and consider the problem,

$$\max_{\tilde{S} \subset T: \; B(\tilde{S}) \cap (D^g \setminus Z^f) \neq \emptyset} V_f(B(\tilde{S}) \cap (D^g \setminus Z^f)), \tag{15}$$

with corresponding solution $\overline{S}$ with $\overline{R} \equiv B(\overline{S}) \cap (D^g \setminus Z^f)$ where $V_f(\overline{R})$ is the value of the objective above. Note that $\overline{R} \subset \text{Supp}(f) \cap \text{Supp}(g)$. Because $D_g \setminus Z^f$ is a feasible S_a in Theorem 2 under f, $V_f(\overline{R}) \geq \pi_f(t|U^R)$. By Lemma 1 V_f is downward biased on $\overline{R}$. Thus by Proposition 3, this means that $V_g(\overline{R}) \geq V_f(\overline{R})$. Now notice that by Proposition 2,

$$\pi_g(t|U^R) = \max_{\tilde{S} \subset T: \; B(\tilde{S}) \cap D^g \neq \emptyset} V_g(B(\tilde{S}) \cap D^g).$$

Moreover, because Z^f is a lower contour subset of T,

$$\max_{\tilde{S} \subset T: \, B(\tilde{S}) \cap D^g \neq \emptyset} V_g(B(\tilde{S}) \cap D^g) \geq \max_{\tilde{S} \subset T: \, B(\tilde{S}) \cap (D^g \setminus Z^f) \neq \emptyset} V_g(B(\tilde{S}) \cap (D^g \setminus Z^f)).$$

Thus $\pi_g(t|U^R) \geq V_g(\overline{R})$. Putting this string of inequalities together gives the desired conclusion that $\pi_f(t|U^R) \leq \pi_g(t|U^R)$.

" $\impliedby$ " Let $t \succeq_d t'$ and $f, g \in \Delta T$ such that $f(t)g(t') < f(t')g(t)$ with $t' \in \text{Supp}(f) \cap \text{Supp}(g)$. Define $S \equiv W(\{t\}) \cap B(\{t'\})$, and $\tilde{S} \equiv S \setminus \{t, t'\}$. I prove the case in which $F(\tilde{S})g(t') \geq G(\tilde{S})f(t')$; the opposite case is symmetric. Consider two actions $\overline{a} > \underline{a}$ and let U^R be quadratic loss, with (i) $v(s) = \overline{a} \; \forall s \notin W(\{t\})$, (ii) $v(s) = \underline{a} \; \forall s \in W(\{t\}) \setminus B(\{t'\})$, (iii) $v(s) = \overline{a} \; \forall s \in S \setminus \{t\}$, and (iv) $v(t) = \underline{a}$.

For any distribution $h \in \Delta T$, the ROE partition is made up of 3 parts given by $P_1 = W(\{t\}) \setminus S$, $P_2 = S$ and $P_3 = T \setminus W(\{t\}))$. The associated actions (as long as each part has positive support are $V_h(P_1) = \underline{a}$, $V_h(P_3) = \overline{a}$, and $V_h(P_2) = V_h(S) = (H(S \setminus \{t\})\overline{a} + h(t)\underline{a})/H(S)$. Note that by assumption both f and g put positive probability on S. $V_f(S) > V_g(S)$ if $\frac{f(t)}{F(\tilde{S} \cup \{t'\})} < \frac{g(t)}{G(\tilde{S} \cup \{t'\})}$ which holds by assumption. Thus $\pi_f(t'|U^R) > \pi_g(t'|U^R)$.

Q.E.D.

C. Proofs from Section 6

C.1. Proof of Proposition 4

Proof. Let $q_e \equiv \frac{p_H q}{p_H q + p_L(1-q)}$ and $q_\emptyset \equiv \frac{(1-p_H)q}{(1-p_H)q + (1-p_L)(1-q)}$ be the update on the sender's type given an evidence type and no evidence respectively in period 1. Let

$$A(p) \equiv \mathbb{E}[\pi_{pf_H + (1-p)f_L}(t|U_2^R)|t \sim h_2(\cdot|s_i)]$$

be the sender's expected action in period 2 given a period 1 evidence type s_i and facing a receiver who believes the sender is the H type with probability p. Note that $h_2(\cdot|s)$ is the same distribution for all evidence types $s \in \{s_1, ..., s_{n-1}\}$so the above is well defined.

Take any candidate period 1 equilibrium strategies with $a_1(s_\emptyset)$ and $p_\emptyset$ being the receiver's best response to non-disclosure and associated belief that the sender is the H type. The sender withholds evidence s_i if $rv_1(s_i) + A(q_e) \leq ra_1(s_\emptyset) + A(p_\emptyset)$. Since the LHS is strictly increasing in i and the RHS is constant, in any equilibrium the sender's period 1 strategy σ is monotonic, i.e. for $i > j$, $\sigma_{s_i}(s_\emptyset) > 0 \implies \sigma_{s_j}(s_\emptyset) = 1$. Note that $p_\emptyset < q_e$ for any sender strategy, and so by Theorem 1 $A(q_e) \leq A(p_\emptyset)$. This means that in any equilibrium strategy in which the sender discloses s_i Corollary 1 implies that $v_1(s_i) \geq a_1(s_\emptyset)$. This means that the equilibrium with the maximum non-disclosure action maximizes the set of evidence types that withhold. I will construct such an equilibrium below.

For a given $k \in [0, n-1]$ define the measure

$$\tilde{h}_k(s) \equiv \begin{cases} h_1(s) & s \in \{s_\emptyset, s_1, ..., s_{\lfloor k \rfloor}\} \\ (k - \lfloor k \rfloor)h_1(s) & s = s_{\lceil k \rceil} \\ 0 & \text{otherwise} \end{cases}$$

and the probability distribution $h_k(s) \equiv \frac{\tilde{h}_k(s)}{\tilde{h}_k(S)}$. Define $p_k \equiv h_k(s_1, ..., s_{n-1})q_e +$

$h_k(s_\emptyset)q_\emptyset$. Define $K_r \equiv \{k \in [0, n-1] : rV^1_{h_k}(S) + A(p_k) \geq rv^1(s_{\lceil k \rceil}) + A(q_e)\}$. Note that K_r is non-empty because $V^1_{h_0}(S) = v_1(s_\emptyset) > v_1(s_1)$ and $A(q_e) \leq A(q_\emptyset)$ by Theorem 1. Let $\overline{k}_r$ be the maximal element of K. Note that $rV^1_{h_k}(S) + A(p_k)$ is continuous in k, so for $\overline{k}_r \notin \mathbb{Z}$, $rV^1_{h_{\overline{k}_r}}(S) + A(p_{\overline{k}_r}) = rv_1(s_{\lceil \overline{k}_r \rceil}) + A(q_e)$. This means we can construct the following equilibrium strategy σ for the sender in period 1:

$$\sigma_{s_i}(s_\emptyset) = \begin{cases} 1 & i < \overline{k}_r \\ \overline{k}_r - \lfloor \overline{k}_r \rfloor & i = \lceil \overline{k}_r \rceil \\ 0 & i > \lceil \overline{k}_r \rceil \end{cases}.$$

By definition, σ for the sender and the receiver choosing $a_1(s_\emptyset) = V^1_{h_{\overline{k}_r}}(S)$ and $a_1(s_i) = v_1(s_i)\ \forall i$ constitutes the equilibrium with the maximum non-disclosure action. Now consider any $k \in [0, n-1]$. If $k \in K_r$ then $rV^1_{h_k}(S) + A(p_k) \geq rv^1(s_{\lceil k \rceil}) + A(q_e)$ or $A(p_k) - A(q_e) \geq r(v^1(s_{\lceil k \rceil}) - V^1_{h_k}(S))$. Since the LHS is positive, if this inequality holds for r, then it also holds for $r' < r$. That is, $K_r \subset K_{r'}$. This means $\overline{k}_r < \overline{k}_{r'}$ completing the argument. *Q.E.D.*

C.2. Proof of Proposition 5

Proof. I first show that the UEPP implies that the receiver does not benefit from early communication. Consider any equilibrium allocation $\tilde{\pi} : T \to \Delta A$ with sender period 1 strategy σ. Denote the receiver's beliefs over T in period 2 following on path declaration s in period 1 as $g_s \in \Delta T$ with corresponding ROE partition of $B(\{t_1 : \sigma_{t_1}(s) > 0\})$ given by $(P^s_1, ..., P^s_{m(s)})$.[39] Note that,

$$g_s(t_2) = \frac{\sum_{t_1 \in W(\{t_2\})} \frac{h_2(t_2)}{H_2(B(t_1))} \sigma_{t_1}(s) h_1(t_1)}{\sum_{t_1 \in T} \sigma_{t_1}(s) h_1(t_1)}. \tag{16}$$

Note that if $\tilde{\pi}$ is degenerate then the the result follows from the fact the degenerate action as a response to each equivalence class must be the best response to that set under the unconditional period 2 distribution by Lemma 2. Suppose that s' and s'' are two on path period 1 declarations. I first show the following claim.

Claim 2. *Let $S \subset T$ be an interval. If $\sigma_{t_1}(s'') = 0\ \forall t_1 \in S$ then $g_{s'} \geq_{ME} g_{s''}$ with respect to $(S, \succeq_d)$.*

[39] This is the space over which there is positive support for period 2 evidence.

Proof of Claim: Take $\bar{t}_2, \underline{t}_2 \in P_k^{s'}$ such that $\bar{t}_2 \succeq_d \underline{t}_2$. Notice that for any period 1 declaration s, $g_s(\bar{t}_2)/h(\bar{t}_2) \geq g_s(\underline{t}_2)/h(\underline{t}_2)$. To see this expand the expressions on either side of the inequality using (16) to get

$$\sum_{t_1 \in W(\bar{t}_2)} \frac{\sigma_{t_1}(s_1'')h_1(t_1)}{H_2(B(t_1))} \geq \sum_{t_1 \in W(\underline{t}_2)} \frac{\sigma_{t_1}(s_1'')h_1(t_1)}{H_2(B(t_1))}.$$

Since $\bar{t}_2 \succeq_d \underline{t}_2$, the LHS sums more terms (in a set containment sense) and is therefore greater than the RHS. By showing that the above is an equality for $s = s''$, we will have shown that $g_{s'}(\bar{t}_2)g_{s''}(\underline{t}_2) \geq g_{s'}(\underline{t}_2)g_{s''}(\bar{t}_2)$, i.e. $g_{s'} \geq_{ME} g_{s''}$ on S. Suppose to the contrary that the LHS>RHS for $s = s''$. This means there exists $t_1 \in W(\{\bar{t}_2\}) \setminus W(\{\underline{t}_2\})$ such that $\sigma_{t_1}(s'') > 0$. But since $t_1, \underline{t}_2 \in W(\bar{t}_2)$ and since $\underline{t}_2 \not\succeq_d t_1$, by the UEPP $t_1 \succeq_d \underline{t}_2$. But because S is an interval, $t_1 \in S$. This means that $\sigma_{t_1}(s'') = 0$ by assumption, which is a contradiction.

I will show that $\forall t \in \text{Supp}(g_{s'}) \cap \text{Supp}(g_{s''})$, we have that (i) $t \in P_k^{s'} \cap P_j^{s''} \implies P_k^{s'} \cap \text{Supp}(g_{s''}) = P_j^{s''} \cap \text{Supp}(g_{s'})$ and (ii) $\pi_{g_{s'}}(t|U^R) = \pi_{g_{s''}}(t|U^R)$. In words, when restricted to their mutual support, both period 1 disclosures induce the same ROE partition and allocation.

Let $P_k^{s'}$ be the highest index part of $P^{s'}$ to violate condition (i) or (ii) or both. Take $P_j^{s''}$ with the highest index j such that $P_j^{s''} \cap P_k^{s'} \neq \emptyset$. This means that $P_{j'}^{s''} \cap P_k^{s'} = \emptyset \ \forall j' > j$ and since higher parts of $P^{s'}$ cannot violate (i), $P_j^{s''} \cap P_{k'}^{s'} = \emptyset \ \forall k' > k$. This means that $P_k^{s'}$ is feasible in the maximization that selects $P_j^{s''}$, and vice versa, in the construction of each partition by Consider constructing $P^{s'}$ through the maximization version of Algorithm 1. At the stage where $P_k^{s'}$ is selected, the fact that $P_j^{s''} \cap P_{k'}^{s'} = \emptyset \ \forall k' > k$ means that $P_j^{s''}$ is also available. The symmetric point for the construction of $P^{s''}$ and $P_k^{s'}$ being available is also true. This means that $V_{g_{s'}}(P_k^{s'}) \geq V_{g_{s'}}(P_j^{s''})$ and $V_{g_{s''}}(P_j^{s''}) \geq V_{g_{s''}}(P_k^{s'})$.

Now suppose that $V_{g_{s'}}(P_k^{s'}) > V_{g_{s''}}(P_j^{s''})$. Each period 1 type $t_1 \in P_k^{s'}$ either expects to remain in $P_k^{s'}$ in period 2 in which case he gets a strictly higher action from s', or to end up in a higher part in which case he is indifferent between s' and s'' by assumption. This means that declaring s' strictly dominates declaring s'' for t_1 and so $\sigma_{t_1}(s'') = 0 \ \forall t_1 \in P_k^{s'}$. Thus by Claim 2 $g_{s'} \geq_{ME} g_{s''}$ on $P_k^{s'}$. Using Proposition 3, this implies that $V_{g_{s'}}(P_k^{s'}) \leq V_{g_{s''}}(P_k^{s'})$. Using the inequality above that $V_{g_{s''}}(P_j^{s''}) \geq V_{g_{s''}}(P_k^{s'})$ leads to a contradiction. The opposite case in which $V_{g_{s'}}(P_k^{s'}) < V_{g_{s''}}(P_j^{s''})$ is symmetric.

The only remaining case is that $V_{g_{s'}}(P_k^{s'}) = V_{g_{s''}}(P_j^{s''})$. In this case $P_k^{s'}$ does not violate condition (ii) so it must violate condition (i) by assumption. Without loss, let $P_k^{s'} \not\subset P_j^{s''}$, the opposite case being symmetric. Now define the non-empty set $R \equiv \cup_{j'<j} P_{j'}^{s'} \cap P_k^{s'}$. Notice that period 1 types in R strictly prefer declaration s' to s'': in period 2 they either remain in R in which case s' is preferred or they end up in a higher part in which case they are indifferent by assumption. This means that $\sigma_{t_1}(s'') = 0 \; \forall t_1 \in R$. Thus by Claim 2 $g_{s'} \geq_{ME} g_{s''}$ on R.

Since by assumption $P_j^{s''} \cap P_{k'}^{s'} = \emptyset \; \forall k' \geq k$, the upper contour subset of $P_{j'}^{s''}$ given by $(\cup_{k' \geq k} P_{k'}^{s'}) \cap P_{j'}^{s''} = P_k^{s'} \cap P_{j'}^{s''}$. Thus by Proposition 1 $V_{g_{s''}}(P_k^{s'} \cap P_{j'}^{s''}) \leq V_{g_{s''}}(P_{j'}^{s''}) < V_{g_{s''}}(P_j^{s''}) \; \forall j' < j$. R is the disjoint union of these upper contour subsets and so by Lemma 2 $V_{g_{s''}}(R) < V_{g_{s''}}(P_j^{s''})$. By analogous logic R is a lower contour subset of $P_k^{s'}$. This means that any lower contour subset of R, denoted $\underline{R}$, is in turn a lower contour subset of $P_k^{s'}$ and thereby has $V_{g_{s'}}(\underline{R}) \geq V_{g_{s'}}(P_k^{s'})$. The following claim extending Proposition 3 establishes a contradiction using $S = R$ and $\overline{v} = V_{g_{s'}}(P_k^{s'})$. This completes the proof.

Claim 3. *Suppose that $(S, \succeq_d)$ is a disclosure ordered subset. Take two distributions $f, g \in \Delta S$ such that $f \geq_{ME} g$. If $\overline{v}$ is such that, $V_f(W(S') \cap S) \geq \overline{v} \; \forall S' \subset S$, then $V_g(S) \geq \overline{v}$.*

Proof of Claim: Suppose not, i.e. $V_g(S) < \overline{v}$. There exists a lower contour subset $\underline{W} \subset S$ such that $V_g(\underline{W}) < \overline{v}$ and $V_g(W(S') \cap \underline{W}) \geq \overline{v} \; \forall S' : \; \underline{W} \not\subset W(S')$. One can find such a set by starting with S and then selecting contained lower contour subsets that have value less than $\overline{v}$ until such a proper lower contour subset does not exist. Notice that by construction $\underline{W}$ is a downward biased set. Thus by Proposition 3 $V_g(\underline{W}) \geq V_f(\underline{W})$ but by assumption $V_f(\underline{W}) \geq \overline{v}$, a contradiction. This completes the proof of the claim.

Now suppose that $(T, \succeq_d)$ does not satisfy the UEPP. This means there exists $t_1, t_2, t_3 \in T$ such that $t_1 \succeq_d t_2$ and $t_1 \succeq_d t_3$ with $t_2 \not\succeq_d t_3$ and $t_3 \not\succeq_d t_2$. Suppose that $\text{Supp}(h_1) = \text{Supp}(h_2) = \{t_1, t_2, t_3\}$. Let U^R be quadratic loss with $v(t_1) = v(t_3) = 0$ and $v(t_2) = 1$. For any distribution $g \in \Delta\{t_1, t_2, t_2\}$ in period 2 the ROE partition is $(\{t_3\}, \{t_1, t_2\})$ with $\pi_g(t_3|U^R) = 0$ and $\pi_g(t_2|U^R) = \pi_g(t_1|U^R) = \frac{g(t_2)}{g(t_1)+g(t_2)}$. The sender's pure period 1 disclosure strategy in which $\sigma_{t_2}(t_2) = \sigma_{t_3}(t_2) = \sigma_{t_3}(t_3) = 1$ is an equilibrium of the dynamic disclosure game $\forall h_1, h_2 \in \Delta\{t_1, t_2, t_3\}$. The induced $\tilde{\pi} : T \to \Delta\mathbb{R}$ is non-degenerate as declaring t_3 in period 1 leads to an action of 0 for t_1 in period 2. But declaring t_2 in period 1 leads to positive action for t_1 in period 2. *Q.E.D.*

D. Proofs from Section 7

D.1. Proof of Proposition 6

Proof. Consider any equilibrium allocation of $\tilde{C}$ given by $\tilde{\pi} : T \to \mathbb{R}$. I will show that $\forall \bar{t}, \underline{t} \in T,\ \bar{t} \succeq_d \underline{t} \implies \tilde{\pi}(\bar{t}) \geq \tilde{\pi}(\underline{t})$. Suppose not. By the definition of $\tilde{\succeq}_d$ there exists $t, t' \in T'$ such that either

$$\tilde{\pi}((t,H)) < \tilde{\pi}((t',H)),\ t \succeq'_d t',\ \text{or} \tag{17}$$

$$\tilde{\pi}((t,L)) > \tilde{\pi}((t',L)),\ t \succeq'_d t',\ \text{or} \tag{18}$$

$$\tilde{\pi}((t,H)) < \tilde{\pi}((t',L)),\ \exists s \in T' :\ t \succeq_d s,\ t' \succeq_d s. \tag{19}$$

The statements in (17) and (18) directly violate sender incentive compatibility in $\tilde{C}$. In (19) sender incentive compatibility in $\tilde{C}$ requires that $\tilde{\pi}((t,H)) \geq \tilde{\pi}((s,H))$ and $\tilde{\pi}((t',L)) \leq \tilde{\pi}((s,L))$, which combined with (19) gives $\tilde{\pi}((s,H)) < \tilde{\pi}((s,L))$ violating sender incentive compatibility in $\tilde{C}$. Because $(T, \succeq_d)$ fits the basic framework, Claim 1 says that the receiver does better with $\pi(t|U^R)$ than $\tilde{\pi}$ with a strict improvement if these are different. Thus all I need to show is that $\pi(t|U^R)$ is an equilibrium allocation in $\tilde{C}$. By Proposition 1 $\pi(t|U^R)$ corresponds to a interval partition of $(T, \succeq_d)$, $(P_1, ..., P_m)$ such that V_h is downward biased on each $(P_i, \succeq_d)$. First I show that there is a pooling strategy on $(P_i, \tilde{\succeq}_d)$. Define

$$\overline{P}_i \equiv \{t \in T' :\ (\{(t,H)\} \cup \{(t,L)\}) \cap P_i \neq \emptyset\},$$
$$\overline{h} \in \Delta P_i :\ \overline{h}(t) \equiv H(\{(t,H)\} \cup \{(t,L)\})/H(P_i),\ \text{and}$$
$$\overline{U}^R(a,t') \equiv \mathbb{E}[U^R(a,t) | t \in (\{(t',H)\} \cup \{(t',L)\}) \cap P_i,\ t \sim h].$$

In words $(\overline{P}_i, \succeq'_d)$ are the partially ordered equivalence classes of $(P_i, \tilde{\succeq}_d)$ and $\overline{h}$ and $\overline{U}^R$ the induced distribution and receiver utility respectively. Let the associated receiver best responses be $\overline{V}_{\overline{h}}$. Note that by construction $\overline{V}_{\overline{h}}(\overline{P}_i) = V_h(P_i)$. Clearly if there is a pooling strategy on $(\overline{P}_i, \succeq'_d)$ then there is a pooling strategy on $(P_i, \tilde{\succeq}_d)$ because one could use the exact same strategy by selecting a single element of each equivalence class. We will verify (12) on $(\overline{P}_i, \succeq'_d)$ which by Lemma 3 implies the existence of a pooling strategy. Consider $W' \subset \underline{W}_{\succeq'_d}(\overline{P}_i)$.

Let $M \equiv W_{\succeq_d}((E_{\succeq'_d}(W') \times \{H, L\})) \cap P_i$. M is a lower contour subset of $(P_i, \succeq_d)$ and so because V_h is downward biased on $(P_i, \succeq_d)$, $V_h(P_i) \leq V_h(M)$. Now take $\overline{M} = \{t \in \overline{P}_i :\ (t,H) \in M \text{ or } (t,L) \in M\}$. By construction $\overline{V}_{\overline{h}}(\overline{M}) = V_h(M)$ and

$E_{\succeq'_d}(W') \subset \overline{M}$. Thus the fact that $Q(W')$ increases the probability of types with higher value than $V_h(P_i)$ and decreases the probability of types with lower value than $V_h(P_i)$ verifies (12).

All that is left to verify is that the $\pi(t|U^R)$ is sender incentive compatible in $\tilde{C}$. Suppose not. By the definition of $\tilde{\succeq}_d$ there exists $t, t' \in T' : t \succeq_d t'$ such that either

$$\pi((t,H)|U^R) < \pi((t',H)|U^R), \text{ or}$$
$$\pi((t,L)|U^R) > \pi((t',H)|U^R), \text{ or}$$
$$\pi((t,H)|U^R) < \pi((t',L)|U^R), \text{ or}$$
$$\pi((t,L)|U^R) > \pi((t',L)|U^R).$$

Note that if $t \succeq'_d t'$ then both $(t,H)\tilde{\succeq}_d(t',L)$ and $(t',H)\tilde{\succeq}_d(t',L)$. This means that $(t,H) \succeq_d (t',H) \succeq_d (t',L) \succeq_d (t,L)$. Thus all the above statements violate sender incentive compatibility of $\pi(t|U^R)$ in $\tilde{D}$. *Q.E.D.*

D.2. Proof of Proposition 7

Proof. Take some ROE pooled set in $\tilde{D}$, P_i. Assume that P_i contains both H and L type senders, i.e. $(t',H) \in P_i$ and $(t'',L) \in P_i$. However, $t' \succeq'_d \underline{s}$ implies $(t',H) \succeq_d (\underline{s},H)$ and $t'' \succeq'_d \underline{s}$ implies $(\underline{s},H) \succeq_d (t'',L)$ by definition. Since P_i is an interval $(\underline{s},H) \in P_i$. The symmetric argument shows that $(\underline{s},L) \in P_i$. Thus at most one pooled set can contain both H and L type senders.

Now assume that there is no set that contains both H and L type senders. Because $\underline{s}$ is a lower bound of T' every H type dominates every L type. This means that every H type is in a higher part of the ROE than every low type. Let $P_1, ..., P_k$ be the ROE pooled sets in $\tilde{D}$ that contain L types, i.e. $P_1 \cup ... \cup P_k = T' \times \{L\}$. Let $P_{k+1}, ..., P_m$ be the ROE pooled sets in $\tilde{D}$ that contain H types, i.e. $P_{k+1} \cup ... \cup P_m = T' \times \{H\}$. Because $\underline{s}$ is a lower bound of T' every H type dominates every L type. By the independence assumptions of the receiver's preferences and Lemma 2, $V_h(T') = V_h(P_1 \cup ... \cup P_k) \leq V_h(P_k)$. Similarly, $V_h(T') = V_h(P_{k+1} \cup ... \cup P_m) \geq V_h(P_{k+1})$. Since $V_h(P_{k+1}) > V_h(P_k)$, this is a contradiction. *Q.E.D.*

D.3. Proof of Proposition 8

Proof. The fact that the expression in (9) corresponds to the ROE in $\tilde{H}$ flows from applying Theorem 2. Consider any strategic type (t, S) and any feasible sets $S_a, S_b \subset$

T in the problem in Theorem 2 for type (t, S). Let the division of evidence types in $W(S_a)$ according to strategic and honest types be $W(S_a) \equiv (W^S \times \{S\}) \cup (W^H \times \{H\})$ for $W^H, W^S \subset T'$. Similarly, let the division of evidence types in $B(S_b)$ according to strategic and honest types be $B(S_b) \equiv (B^S \times \{S\}) \cup (B^H \times \{H\})$ for $B^H, B^S \subset T'$. Notice that because honest types only dominate themselves and are dominated by their strategic counterpart $W^S \subset W^H$ and $B^H \subset B^S$. Thus,

$$\begin{aligned} W(S_a) \cap B(S_b) &= ((W^S \cap B^S) \times \{S\}) \cup (((W^S \cap B^H) \cup ((W^H \setminus W^S) \cap B^H)) \times \{H\}) \\ &\equiv M(W^H, W^S, B^H, B^S). \end{aligned}$$

Let UCS and LCS be the sets of upper and lower contour subsets of T'. We can rewrite the problem in Theorem 2 as $\pi_h(t|U^R) =$

$$\min_{W^S \subset LCS\colon t \in W^S, \text{ and } W^H\colon W^S \subset W^H} \; \max_{B^S \subset UCS\colon t \in B^S, \text{ and } B^H \subset B^S} V_h(M(W^H, W^S, B^H, B^S))$$

Note that it is without loss to choose $W^H = W^S$. If not then it must be that $(W^H \setminus W^S) \cap \overline{B}^H \neq \emptyset$ where $\overline{B}^H, \overline{B}^S$ is the solution to the partial maximization given some W^H, W_L. But then the $V_{f^{NS}}((W^H \setminus W^S) \cap \overline{B}^H) \geq V_h(M(W^H, W^S, \overline{B}^H, \overline{B}^S))$ otherwise the maximizer would do better setting $B^H \cap W^S \setminus W^H = \emptyset$. But this means that the minimizer weakly decreases the value in the problem above by setting $W^H = W^S$. Thus we can rewrite the problem as,

$$\begin{aligned} &\min_{W^S \subset LCS\colon t \in W^S} \; \max_{B^S \subset UCS\colon t \in B^S, \text{ and } B^H \subset B^S} V_h(((W^S \cap B^S) \times \{S\}) \cup ((W^S \cap B^H) \times \{H\})) \\ &= \min_{W^S \subset LCS\colon t \in W^S} \; \max_{B^S \subset UCS\colon t \in B^S} \tilde{V}(W^S \cap B^S), \end{aligned}$$

where the equality comes from the definition of $\tilde{V}$ in (8). This means that replacing V_h with $\tilde{V}$ solves the problem in Theorem 2 for strategic types as in (9). Let $\overline{B}^S$ and $\overline{B}^H$ solve the above problem for optimal $\underline{W}^S$. This means that the honest types in $(\overline{B}^S \setminus \overline{B}^H) \times \{H\}$ cannot be used in any strategic type's optimization. Combined with the fact that the honest types cannot mimic other honest types, this means that $(\overline{B}^S \setminus \overline{B}^H) \times \{H\}$ must fully separate, i.e. $\forall t' \in \overline{B}^S \setminus \overline{B}^H \; \pi((t', H)|U^R) = v(t')$. This would only be incentive compatible if their value were less than the action obtained by their strategic counterparts. This shows that the characterization of π^* is correct for the honest types model.

The strategies for the sender and receiver used in $\tilde{H}$ can be directly imported to $\tilde{U}B$ because the sender has strictly more messaging options in $\tilde{U}B$. The reason that π^* is an equilibrium allocation of $\tilde{UB}$ is as follows. Receiver and strategic sender incentive compatibility are directly transferred from that in $\tilde{H}$ and unbiased sender's who obtain $v(t)$ are at their bliss point. Unbiased senders who obtain $\pi^*(t, NS) < v(t)$ can only potentially deviate to lower actions which are further from their bliss point, otherwise their strategic counterparts would deviate in $\tilde{H}$. Thus π^* is an equilibrium allocation in $\tilde{UB}$.

Now I show that π^* is receiver optimal in $\tilde{UB}$. Consider some other equilibrium allocation π' in $\tilde{UB}$. Define the following alternative allocation $\hat{\pi}$ in $\tilde{UB}$ defined by $\forall t \in T'$:

$$\hat{\pi}((t, S)) = \pi'((t, S))$$
$$\hat{\pi}((t, UB)) = \min\{\pi'((t, UB)), v(t)\}.$$

I first show that $\hat{\pi}$ is sender incentive compatible. Take any $s, s' \in T' \times \{S, UB\}$ such that $s \succeq_d^{UB} s'$ (otherwise IC has no bite). If s is strategic then $\hat{\pi}(s) = \pi'(s) \geq \pi'(s') \geq \hat{\pi}(s')$ so $\hat{\pi}$ is incentive compatible for strategic senders. If s is unbiased, then either $\hat{\pi}(s) = v(s)$, or $v(s) > \hat{\pi}(s) = \pi'(s) \geq \pi'(s') \geq \hat{\pi}(s')$ so $\hat{\pi}$ is also incentive compatible for unbiased senders. The receiver is clearly better off under $\hat{\pi}$ than π' because the only shift is that he now gets his bliss point for some unbiased types. Also $\hat{\pi}$ is equivalent to π' only if $\pi' = \pi^*$. Since $s \succeq_d s'$ in $\tilde{H}$ implies that $\hat{\pi}(s) \geq \hat{\pi}(s')$. Claim 1 completes the proof. *Q.E.D.*

D.4. Proof of Theorem 3

Proof. I only prove that more evidence implies more skepticism. The $\Longleftarrow$ direction is equivalent to that for Theorem 1.

" $\Longleftarrow$ "

Take the ROE partition under $\succeq_d$, $P = (P_1, ..., P_m)$. I show that P is also the ROE partition under $\succeq_{d,v}$ for any $h \in \Delta T$. The result then follows from Theorem 1. Since $\succeq_{d,v}$ is coarser than $\succeq_d$, P remains an interval partition. Thus by Proposition 1 all that remains is to check that V_h is downward biased on $(P_i, \succeq_{d,v})$ $\forall i$.

Suppose not. Take a lower contour subset R of $(P_i, \succeq_{d,v})$ such that $V_h(R) < V_h(P_i)$. It is without loss to assume that this set is of the form $R = W_{\succeq_{d,v}}(\underline{R}) \cap P_i$ where $v(t) < V_h(P_i)$ $\forall t \in \underline{R}$. Since V_h is downward biased on $(P_i, \succeq_d)$, it must be that $V_h(W_{\succeq_d}(\underline{R}) \cap P_i) \geq V_h(P_i)$. This means that there exists $t \in \underline{R}$, $t' \in P_i \setminus R$ such that $t \succeq_d t'$ and $v(t') > V_h(P_i)$. But this means that $v(t') \geq v(t)$ which implies $\succeq_{d,v}$

could not have deleted the dominance between t and t'. Thus $t \succeq_{d,v} t'$ contradicting that R is a lower contour subset of $(P_i, \succeq_{d,v})$.

Q.E.D.

Supplementary Appendix

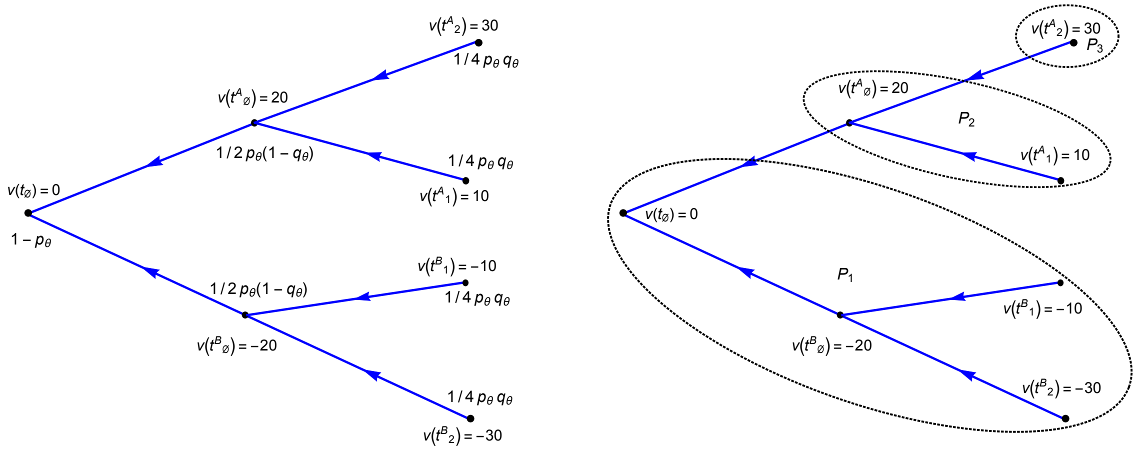
E. Variable Signaling Incentives in Disclosure Games

Consider an arbitrary disclosure game given by $(T, \succeq_d)$. Theorem 1 establishes that a sender with any ex-ante distribution prefers a receiver who believes he is $f_L \in \Delta T$ vs. $f_H \in \Delta T$ where $f_H \geq_{ME} f_L$. Grubb (2011) shows that if $(T, \succeq_d)$ is Dye evidence then the sender with ex-ante distribution f_L prefers this reputation *more* than the sender with ex-ante distribution f_H. The reason is that in the Dye evidence model, the distribution only affects the non-disclosure action which the f_H distribution avoids with higher probability than the f_L distribution. This fact allows for informative signaling in a repeated disclosure context. This section shows that this signaling incentive can actually be reversed for multidimensional evidence structures.

Example 3. Let $T \equiv \{t^A_\emptyset, t^A_1, ..., t^A_{n-1}\} \cup \{t^B_\emptyset, t^B_1, ..., t^B_{n-1}\} \cup \{t_\emptyset\}$, i.e. T is two copies – $\{A, B\}$ – of Dye evidence in combination with an additional no evidence type. The disclosure order $\succeq_d$ is the standard Dye evidence order on each copy, combined with the property that $t \succeq_d t_\emptyset \ \forall t \in T$, and no other comparisons. Let U^R be quadratic loss. Figure 9 shows an example in which $n = 3$ and displays the probability distribution where $\theta \in \{H, L\}$. The idea is that the first piece of evidence is collected with probability p_θ, the second piece is collected with probability q_θ, and given evidence has realized it has high or low value with probability $1/2$. The receiver's best response to each type is displayed next to each evidence type.

Naturally if $p_H > p_L$ and $q_H > q_L$ then the H type has more evidence than the L type. Suppose $q_H = 1/2, \ q_L = 1/8, \ p_H = 1/8$ and consider two alternatives for p_L – either $7/64$ or $1/16$. In both cases, the equilibrium pooling is displayed in Figure 9. In the case in which $p_L = 7/64$ the L type has higher signaling incentives than the H type, i.e. $\mathbb{E}[\pi_{f_L}(t|U^R) - \pi_{f_H}(t|U^R)|t \sim f_L] - \mathbb{E}[\pi_{f_L}(t|U^R) - \pi_{f_H}(t|U^R)|t \sim f_H] > 0$. However if $p_L = 1/16$ then these signaling incentives are reversed, i.e. $\mathbb{E}[\pi_{f_L}(t|U^R) - \pi_{f_H}(t|U^R)|t \sim f_L] - \mathbb{E}[\pi_{f_L}(t|U^R) - \pi_{f_H}(t|U^R)|t \sim f_H] < 0$. The values of both P_1 and P_2 are lower under f_H than f_L. The reason for the ambiguity in signaling incentive is that the H type has a higher chance of reaching P_2 where the L type has a higher chance of being in P_1. The fixed nature of signaling incentives in the Dye model is therefore an artifact of there only being one pooled set.

△



(a) Multidimensional Dye Evidence

(b) Pooling under f_H and f_L

Figure 9: Variable Signaling Incentives

F. Informative Dynamic Signaling Without the UEPP

The following provides an example in which the disclosure order violates the UEPP and the receiver benefits from early communication with the sender.

Example 4. The type space is $\{1, 2, 3, 4, 5, 6\}$ with U^R as quadratic loss, and with the value of each type v_i and $\succeq_d$ both illustrated in the left panel of Figure 10. Notice that the UEPP does not hold as types 3 and 2 are not ordered but are both dominated by type 5. The right panel shows the ROE partition over types under any distribution in period 2. More specifically, types $1, 3, 6$ declare 1, types $2, 5$ declare 2, and type 4 declares 4.

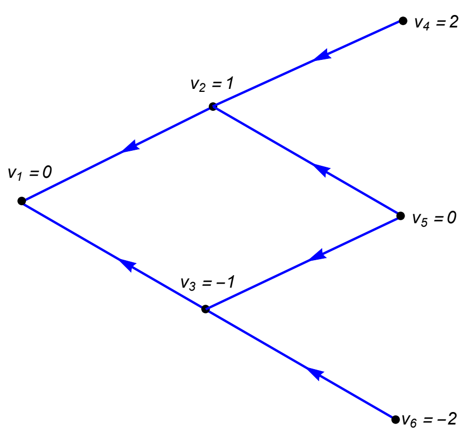
Consider also using the strategy in the right panel of Figure 10 in period 1. Note that 4 truthfully reveals under all period 1 declarations and is thereby indifferent across them. Thus, the only incentive to check is that types 2 and 5 do not want to deviate to declare type 1 in period 1. Let $g_i \in \Delta T$ be the receiver's period 2 belief in ΔT following declaration i in period 1. Types 2 and 5 do not want to deviate if $V_{g_1}(\{2, 5\} \leq V_{g_2}(\{2, 5\})$. Notice that conditional on P_2 type 3 can only give rise to type 4 in period 2. Thus the presence of type 3–a type not in P_2– in a period 1 disclosure lowers the value of P_2 following this period 1 disclosure. This would not be possible under the UEPP.

Incentive compatibility comes down to comparing the likelihood ratio between 2 and 5 in period 2 under the two period 1 declarations. That is, IC holds if

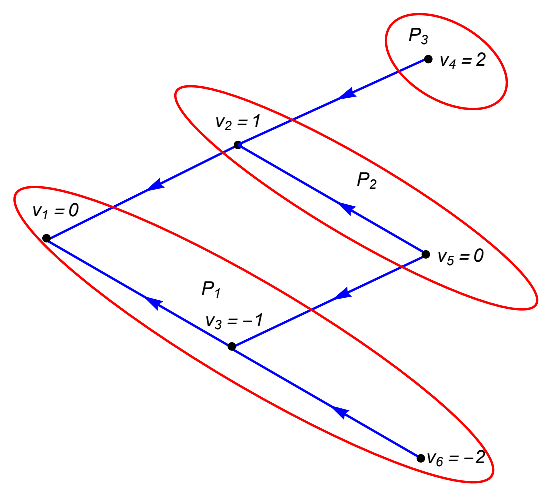
$\frac{g_2(2)}{g_2(5)} \geq \frac{g_1(2)}{g_1(5)}$. If this inequality is strict then the equilibrium is an example where the receiver benefits from early inspections. This inequality is written below in terms of the primitives.

$$\underbrace{\frac{h_2(2)}{h_2(5)} \frac{\frac{h_1(2)}{H_2(\{2,4,5\})}}{\frac{h_1(2)}{H_2(\{2,4,5\})} + \frac{h_1(5)}{H_2(\{5\})}}}_{\text{LR under disclosure } 2} > \underbrace{\frac{h_2(2)}{h_2(5)} \frac{\frac{h_1(1)}{H_2(\{1,\ldots,6\}}}{\frac{h_1(1)}{H_2(\{1,\ldots,6\}} + \frac{h_1(3)}{H_2(\{3,5,6\})}}}_{\text{LR under disclosure } 1}.$$

An example that satisfies this inequality is when h_1 and h_2 are the uniform distribution. Since the receiver's behavior is sequentially optimal and the period 2 equilibrium vector is different following the two on path messages in period 1, the receiver benefits from early inspections. △



(a) $(T, \succeq_d)$ does not satisfy the UEPP



(b) Equilibrium Pooling in Periods 1 and 2

Figure 10: Informative Dynamic Signaling Without the UEPP